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SIMPLIFIED RELIABILITY-BASED DESIGN FOR AXIAL CAPACITY OF FOOTINGS IN COHESIONLESS SOILS — APPLICATION OF THE QUANTILE VALUE METHOD

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ABSTRACT

In this paper, we apply the quantile value method (QVM) to reliability-based design (RBD) for the axial capacities of footings in cohesionless soils. QVM was developed by the first author and his colleague in 2011. It is a new calibration method for simplified RBD codes, and it is proven to be more robust than existing calibration methods, such as the first-order reliability method. A QVM-based code calibration process for footings in cohesionless soils is presented. The calibration is fairly realistic because it incorporates a real footing database recently collected by Akbas and Kulhawy in 2009. Also, a wide range of design scenarios are considered during the calibration. Both the ultimate and serviceability limit states are considered. The calibration results are design charts that can be easily implemented by practicing engineers. Design examples are presented to demonstrate how to implement the design charts. It is shown that designs based on these charts can effectively achieve the target reliability index.

Key words: Reliability-based design, footing, axial capacity, design codes, cohesionless soils.

1. INTRODUCTION

Reliability-based design (RBD) is the basis for many modern geotechnical design codes, such as Eurocode 7 (British Standard Institute 2004) and AASHTO (AASHTO 1997). RBD can be performed with any reliability method, such as Monte Carlo simulation (*e.g.*, Wang 2011) and the first-order reliability method (FORM) (Hasofer and Lind 1974). However, geotechnical engineers typically do not have the knowledge for these reliability methods. As a result, simplified RBD methods are adopted in most design codes. The goal for simplified RBD is to achieve a target reliability level by adopting a set of partial factors (or load/resistance factors). Most codes adopt constant partial factors. For instance, in Annex a of Eurocode 7 British Standard (British Standard Institute 2004), a constant partial factor of 1.4 is recommended for undrained shear strength (s_u) of clays, meaning that (s_u design value) = (s_u characteristic value)/1.4, regardless of how many site investigation efforts are made. Ideally, these partial factors are calibrated by the code developers using reliability methods. There are several reliability methods, and those based on FORM are the most popular.

Ching and Phoon (2011, 2013) pointed out several important shortcomings of design codes based on constant partial factors. The most important shortcoming lies in the fact that such RBD codes cannot produce design outcomes with uniform reliability indices. To speak plainly, two different designs following the same constant partial factors can have dramatically different reliability indices (or failure probabilities). To exemplify this, consider the following two cases: (a) a case with very sophisticated site investigations with many undisturbed clay samples, and their s_u values are determined by triaxial tests; (b) a case with only cursory site investigation with only SPT tests, and s_u values are estimated by the SPT-N values. Case (a) is with less uncertainty because s_u is directly measured, whereas Case (b) is quite uncertain because s_u is indirectly estimated through SPT-N values. Adopting the partial factor of 1.4 for Case (a) may produce a very conservative design (high reliability index), whereas adopting the same partial factor for Case (b) may produce an unconservative design. Ching *et al*. (2014) further illustrated that such RBD codes cannot effectively link site investigation efforts to final design savings. For the above example, Case (a) costs higher than Case (b) in the site investigation. However, the same 1.4 partial factor is adopted for both cases. The final design dimensions for these two cases may be comparable. This is deemed unreasonable. Ching and Phoon (2011, 2013) proposed a new simplified RBD calibration method, called the quantile value method (QVM). They showed that QVM can produce design outcomes with more uniform reliability indices. Ching *et al*. (2014) further illustrated that QVM is able to effectively link site investigation efforts to final design savings.

In this current paper, QVM is used to calibrate the simplified RBD codes for the axial capacity of a footing in a cohesionless soil (sand or gravel). Both the ultimate and serviceability limit states are considered. The calibration is quite realistic in the following sense:

1. A broad range of design scenarios is considered. For instance, the coefficient of variation (COV) of the effective friction angle (ϕ') ranges from 0.05 to 0.2. The width of the footing ranges from 0.2 m to 6 m. The broad range is used to cover most practical design cases.
2. A real footing database was collected by Akbas and Kulhawy (2009a, 2009b). The actual capacities and settle-

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ments for the cases in this database are measured in situ. As a result, the modeling errors are known. The modeling error quantifies the discrepancy between predictive model and reality. These known model errors are used to derive their probability distribution, and this probability distribution is considered during the QVM calibration. The calibrated RBD codes thus incorporate the effect of the modeling error.

2. ULTIMATE LIMIT STATE FOR A FOOTING IN A COHESIONLESS SOIL

This section briefly reviews the predictive equation adopted in this paper for the ultimate axial capacity of a footing in a cohesionless soil. It is assumed that the footing has no base tilt and the ground surface is flat (no sloping). This equation was based on the works by Vesić (1975) and Hansen (1970) and was later improved by Kulhawy *et al.* (1983):

$$\begin{aligned} Q_u &= q_u \cdot A \\ q_u &= 0.5 \cdot B \cdot \gamma_{eff} \cdot N_\gamma \cdot \zeta_{\gamma s}\, \zeta_{\gamma d}\, \zeta_{\gamma r}\, \zeta_{\gamma i} \\ &\quad + \gamma_{eff} \cdot D \cdot N_q \cdot \zeta_{qs}\, \zeta_{qd}\, \zeta_{qr}\, \zeta_{qi} \end{aligned} \tag{1}$$

where Q_u is the (calculated) axial ultimate capacity; q_u is the (calculated) ultimate bearing stress; $A = B \times L$ is the footing area (B is the width, and L is the length); D is the embedment depth of the footing; γ_{eff} is the effective soil unit weight; ϕ' is the effective friction angle; N_γ and N_q are the bearing capacity factors:

$$\begin{aligned} N_q &= \exp\left[\pi \tan(\phi')\right] \tan^2(45 + \phi'/2) \\ N_\gamma &= 2(N_q + 1)\tan(\phi') \end{aligned} \tag{2}$$

The ζ_γ and ζ_q factors are the modifiers. The second subscript indicates the aspect of modification: '*s*' for shape, '*d*' for embedment depth, '*r*' for soil rigidity, and '*i*' for inclination.

$$\begin{aligned} &\zeta_{\gamma s} = 1 - 0.4(B/L) \qquad \zeta_{qs} = 1 + (B/L)\tan(\phi') \\ &\zeta_{\gamma d} = 1 \\ &\zeta_{qd} = 1 + 2\tan(\phi')\left[1 - \sin(\phi')\right]^2 \left[(\pi/180)\tan^{-1}(D/B)\right] \\ &\zeta_{\gamma r} = \zeta_{qr} \\ &\zeta_{qr} = \exp\left\{\left[-4.4 + 0.6(B/L)\right]\tan(\phi') + \frac{3.07\sin(\phi')\log(2I_{rr})}{\left[1 + \sin(\phi')\right]}\right\} \\ &\zeta_{\gamma i} = (1 - \beta/\phi')^2 \qquad \zeta_{qi} = (1 - \beta°/90°)^2 \end{aligned} \tag{3}$$

β is the inclination angle of the total load; $I_{rr} = I_r / (1 + I_r\Delta)$ is the reduced rigidity index (I_r is the rigidity index, and Δ is the volumetric strain):

$$\begin{aligned} I_r &= \frac{E}{2(1+\nu)\, q_{eff}\, \tan(\phi')} \\ \Delta &\approx 0.005\left[(45^o - \phi')/20°\right] \cdot \frac{q_{eff}}{P_a} \end{aligned} \tag{4}$$

q_{eff} is the vertical effective stress at the depth of $B/2$ below the footing; E and ν are the Young's modulus and Poisson ratio of the cohesionless soil (ν is taken to be 0.3); P_a = 101.3 kN/m^2 is one atmosphere pressure. Once I_{rr} is computed, it is compared to the critical rigidity index (I_{rc}):

$$I_{rc} = 0.5\exp\left[(3.3 - 0.45\, B/L)\cot(45 - \phi'/2)\right] \tag{5}$$

If $I_{rr} > I_{rc}$, the footing failure will be in the general shear failure mode. In this case, $\zeta_{\gamma r} = \zeta_{qr} = 1$. Otherwise, ($\zeta_{\gamma r}$, ζ_{qr}) should be calculated by Eq. (3).

Depending on the depth of the ground water table, γ_{eff} and q_{eff} can be calculated as

$$\begin{aligned} &\text{If } h = 0 \quad \gamma_{eff} = \gamma_{sat} - \gamma_w \\ &\qquad q_{eff} = (D + B/2)(\gamma_{sat} - \gamma_w) \\ &\text{If } 0 < h \le D \quad \gamma_{eff} = \gamma_{sat} - \gamma_w \\ &\qquad q_{eff} = h \cdot \gamma_{dry} + (D + B/2 - h)(\gamma_{sat} - \gamma_w) \\ &\text{If } D < h \le D + B \quad \gamma_{eff} = \gamma_{sat} - \gamma_w + \left[(h - D)/B\right] \cdot \gamma_{dry} \\ &\qquad q_{eff} = h \cdot \gamma_{dry} + (D + B/2 - h)(\gamma_{sat} - \gamma_w) \\ &\text{If } h > D + B \quad \gamma_{eff} = \gamma_{dry} \\ &\qquad q_{eff} = (D + B/2)\gamma_{dry} \end{aligned} \tag{6}$$

where h is the depth of the ground water table; γ_w = 9.8 kN/m^3 is the unit weight of water; γ_{sat} and γ_{dry} are the saturated and dry unit weights, respectively:

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1 + e} \qquad \gamma_{dry} = \frac{G_s \gamma_w}{1 + e} \tag{7}$$

G_s is the specific gravity, and e is the void ratio of the cohesionless soil.

Akbas and Kulhawy (2009a, 2009b) collected a database of real footing tests. For all cases, the actual Q_u, denoted by $Q_{u(\text{act})}$, is measured using the $L2$ criterion (Hirany and Kulhawy 1988). The Q_u calculated by Eqs. (1) ~ (7) is denoted by $Q_{u(\text{cal})}$. Figure 1(a) shows the relationship between $Q_{u(\text{act})}$ and $Q_{u(\text{cal})}$. It is found that the following regression equation fits well:

$$\ln(Q_{u(\text{act})} \text{ in kN}) = 1.384 + 0.805\ \ln(Q_{u(\text{cal})} \text{ in kN}) + \varepsilon_Q \tag{8}$$

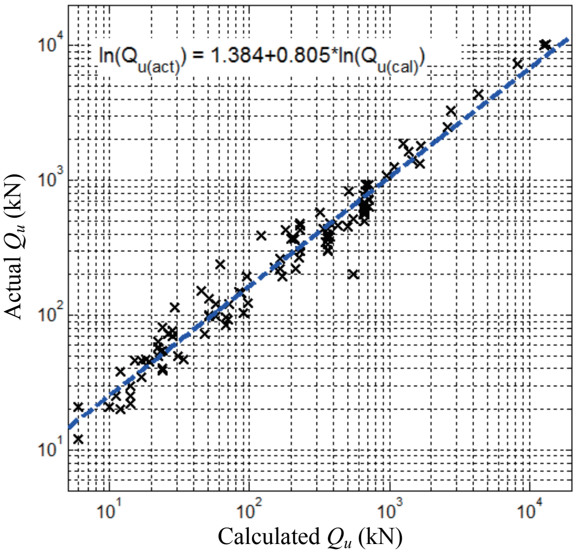
where ε_Q is the zero-mean modeling error term with standard deviation (STD) equal to 0.29. It is modeled as a zero-mean normal random variable with STD equal to 0.29. The same database contains a pairwise (E, ϕ') dataset, shown in Fig. 1(b). It is found that the following regression equation fits well:

$$\ln(E \text{ in kN/m}^2) = 5.785 + 0.101(\phi' \text{ in degree}) + \varepsilon_E \tag{9}$$

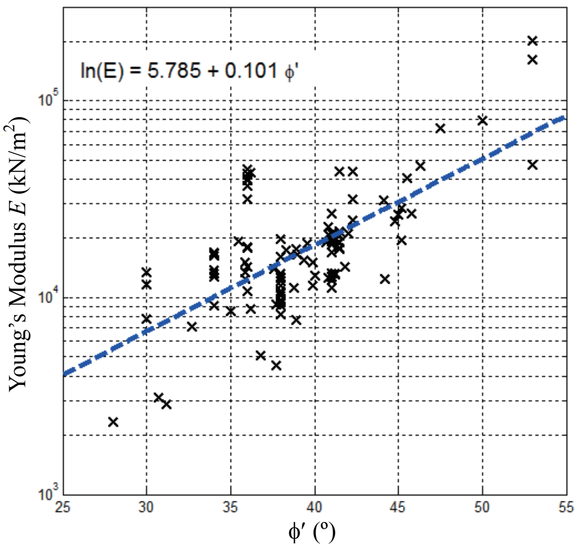
where ε_E is the zero-mean transformation error term with STD equal to 0.51. It is modeled as a zero-mean normal random variable with STD equal to 0.51.

Consider the footing to be subjected to vertical dead load (DL) and vertical live load (LL). The performance function for the ultimate limit state (ULS) is

$$\begin{aligned} G &= Q_{u(\text{act})} - DL - LL - W \\ &= \exp\left[1.384 + 0.805\ln(Q_{u(\text{cal})}) + \varepsilon_Q\right] - DL - LL - W \end{aligned} \tag{10}$$



(a) The relationship between $Q_{u(\text{act})}$ and $Q_{u(\text{cal})}$



(b) The pairwise (E, ϕ') dataset

Fig. 1 The data points collected by Akbas and Kulhawy (2009a, 2009b)

where W is the total weight of the footing, including the soil above the footing foundation. W is conservatively estimated to be $\gamma_c \times D \times B \times L$, where γ_c = 25 kN/m^3 is the unit weight of the concrete.

2.1 Random Variables

The basic random variables for the ULS performance function G include (ϕ', e, DL, LL, ε_Q, ε_E). The Young's modulus E is represented by (ϕ', ε_E) in Eq. (9), so it is not a basic random variable. The Poisson ratio ν = 0.3 is assumed to be a fixed number because the variability of ν is small, so it is not a basic random variable, either. The parameters (B, B/L, D, h, G_s) are deterministic: (B, B/L, D) are decided by the design engineer, whereas (h, G_s) can be measured with negligible uncertainties. Table 1 shows the assumed probability distributions for the six random variables. Except that (ϕ', e) are expected to be negatively correlated, all other random variables are assumed mutually independent. The correlation coefficient between ln(ϕ') and ln(e) is denoted by ρ ($\rho < 0$).

Table 1 The assumed probability distributions for (ϕ', e, DL, LL, ε_Q, ε_E)

Variable	Description	Distribution	Statistics	
			Mean	COV/STD
e	Void ratio	Lognormal	μ_e	COV = V_e
ϕ'	Effective friction angle	Lognormal	μ_ϕ	COV = V_ϕ
DL	Dead load	Gaussian	μ_{DL}	COV = 0.1
LL	Live load	Gumbel	$\mu_{DL} \times r_{L/D}$	COV = 0.2
ε_E	Transformation error for E	Gaussian	0	STD = 0.51
ε_Q	Model error for Q_u	Gaussian	0	STD = 0.29

* COV denotes coefficient of variation; STD denotes standard deviation; V_e denotes COV of e; V_ϕ denotes COV of ϕ'; μ_{DL} denotes mean value of DL; $r_{L/D}$ denotes the ratio between live load mean to dead load mean

2.2 Calibration of QVM

Ching and Phoon (2011, 2013) developed a quantile-based simplified RBD method which is more robust than the simplified RBD method based on constant partial factors (PF). This quantile-based method was referred to as the quantile value method (QVM) in Ching and Phoon (2013). They showed that the PF-based simplified RBD method is not robust under variable coefficients of variation (COV) of soil parameters, but the QVM is. The COV of a soil parameter is not a constant. It depends on how the soil parameter is evaluated and can vary in a wide range (*e.g.*, COV ranges from 0.1 to 0.7 for the undrained shear strength of a clay). The basic idea of the QVM is to reduce any stabilizing random variable (*e.g.*, soil strength) to its η quantile (η is small) to obtain its design value, and to increase any destabilizing random variable (*e.g.*, load) to its $1 - \eta$ quantile to obtain its design value. The parameter η is called the probability threshold, and a constant η is applied to both types of random variables: taking η quantiles for stabilizing variables and $1 - \eta$ quantiles for destabilizing variables. For the above footing ULS problem, the following decision is made regarding the stabilizing/destabilizing random variables: (ϕ', ε_Q, ε_E) are stabilizing, so they are reduced to their η quantiles; (e, DL, LL) are destabilizing, so they are increased to their $1 - \eta$ quantiles.

The η value that achieves a target reliability index (β_T) must be calibrated using reliability theory. Such a reliability theory was developed in Ching and Phoon (2011). They also suggested that a large number of "calibration cases" should be adopted to calibrate η. It is desirable that these calibration cases are sufficiently different. In this paper, 100 calibration cases with different (B, B/L, D, h, G_s, μ_e, V_e, μ_ϕ, V_ϕ, μ_{DL}, $r_{L/D}$) are randomly sampled from the rectangular volume formed by the ranges summarized in Table 2. For ρ [correlation between ln(e) and ln(ϕ')], four

Table 2 The ranges for (B, B/L, D, h, G_s, μ_e, V_e, μ_ϕ, V_ϕ, μ_{DL}, $r_{L/D}$, ρ)

Parameter	Description	Range
B	Footing width	[0.2 m ~ 6 m]
B/L	Width to length ratio	[0.3 ~ 1]
D	Embedment depth	[0 m ~ 2 m]
h	Depth of the ground water table	[0 m ~ 10 m]
G_s	Specific gravity	[2.6 ~ 2.9]
μ_e	Mean value of e	[0.25 ~ 0.8]
V_e	COV of e	[0.1 ~ 0.3]
μ_ϕ	Mean value of ϕ'	[30° ~ 50°]
V_ϕ	COV of ϕ'	[0.05 ~ 0.15]
μ_{DL}	Mean value of DL	Determined by FS
$r_{L/D}$	Ratio between live load and dead load means	[0.1 ~ 1]
ρ	Correlation coefficient between ln(e) and ln(ϕ')	−0.8, −0.5, −0.3, 0

discrete values are consider: 0, −0.3, −0.5, and −0.8. For each calibration case, μ_{DL} is randomly chosen so that the resulting factor of safety (FS) is between 3 and 6, where FS is defined as

$$FS = \frac{\exp\left[1.384 + 0.805\ \ln(Q_{u(\text{cal}),n})\right] - W}{(1 + r_{L/D})\mu_{DL}} \tag{11}$$

where $Q_{u(\text{cal}),n}$ is the nominal value of $Q_{u(\text{cal})}$ evaluated based on $\phi' = \mu_\phi$, $e = \mu_e$, $\varepsilon_Q = 0$, and $\varepsilon_E = 0$. These 100 calibration cases should cover a wide variety of future design cases because the ranges in Table 2 are fairly wide. Given a target reliability index (β_T), the required η value is calibrated for each of the 100 calibration cases using the theory developed in Ching and Phoon (2011). The average of these 100 η values is then the final η value adopted in QVM. Figure 2 shows how the average η value changes with β_T under various ρ values.

2.3 Design Illustration

With the η versus β_T relationship in Fig. 2, simplified RBD based on QVM can be conducted. To illustrate the design process, a future design scenario with μ_{DL} = 1000 kN, B/L = 1, D = 1 m, h = 0 m, G_s = 2.7, μ_e = 0.4, V_e = 0.2, μ_ϕ = 35°, V_ϕ = 0.1, $r_{L/D}$ = 0.5, and ρ = −0.5 is considered. The goal is to design the width B such that the target reliability index (β_T) is equal to 3.2. The design process is iterative. Let us start with B = 3 m. For β_T = 3.2, the corresponding η value is equal to 0.0246 (see Fig. 2). Recall that (ϕ', ε_Q, ε_E) are stabilizing random variables, the design engineer should take the 0.0246 quantiles of (ϕ', ε_Q, ε_E) as their design values, denoted by (ϕ'_d, $\varepsilon_{Q,d}$, $\varepsilon_{E,d}$). Because ϕ' is lognormal, we have

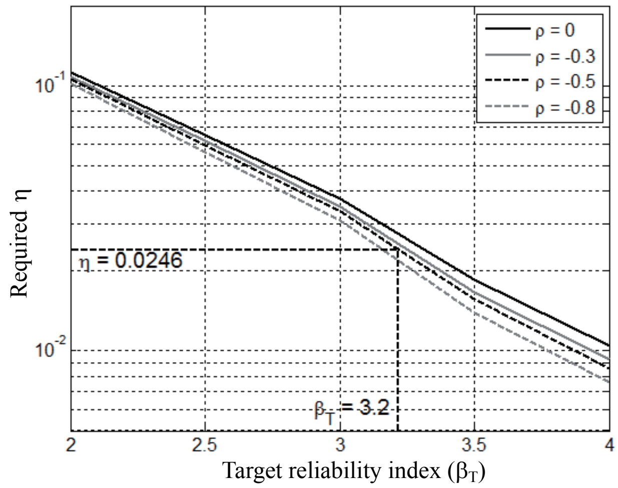


Fig. 2 The relationship between η and β_T under various ρ values (ULS)

$$\phi'_d = \exp\left[\ln\left(\mu_\phi / \sqrt{1+V_\phi^2}\right) + \sqrt{\ln(1+V_\phi^2)} \cdot \Phi^{-1}(\eta)\right] = 28.62° \tag{12}$$

whereas (ε_Q, ε_E) are zero-mean normal with STD = 0.29 and 0.51, respectively, we have

$$\varepsilon_{Q,d} = 0.29 \cdot \Phi^{-1}(\eta) = -0.57 \quad \varepsilon_{E,d} = 0.51 \cdot \Phi^{-1}(\eta) = -1.00 \tag{13}$$

(e, DL, LL) are destabilizing random variables, the design engineer should take the 1 − 0.0246 = 0.9754 quantiles of (e, DL, LL) as their design values, denoted by (e_d, DL_d, LL_d). Because e is lognormal, we have

$$e_d = \exp\left[\ln\left(\mu_e / \sqrt{1+V_e^2}\right) + \sqrt{\ln(1+V_e^2)} \cdot \Phi^{-1}(1-\eta)\right] = 0.266 \tag{14}$$

whereas DL is normal with COV = 0.1, we have

$$DL_d = \mu_{DL} + 0.1 \cdot \mu_{DL} \cdot \Phi^{-1}(1-\eta) = 1196.63 \text{ kN} \tag{15}$$

LL is Gumbel with COV = 0.2, we have

$$\begin{aligned} LL_d &= \alpha_{LL} - \beta_{LL} \ln\left[-\ln(1-\eta)\right] = 742.80 \text{ kN} \\ &\left(\beta_{LL} = 0.2 \cdot \mu_{LL} \cdot \sqrt{6}/\pi = 0.2 \cdot r_{L/D} \cdot \mu_{DL} \cdot \sqrt{6}/\pi\right) \\ &(\alpha_{LL} = \mu_{LL} - 0.5772 \cdot \beta_{LL} = r_{L/D} \cdot \mu_{DL} - 0.5772 \cdot \beta_{LL}) \end{aligned} \tag{16}$$

Based on the design values (ϕ'_d, $\varepsilon_{Q,d}$, $\varepsilon_{E,d}$, e_d, DL_d, LL_d), the design engineer can further calculate the design values for the Young's modulus (denoted by E_d) using Eq. (9), the design values for γ_{sat} and γ_{dry} (denoted by $\gamma_{sat,d}$ and $\gamma_{dry,d}$) using Eq. (7):

$$E_d = \exp(5.785 + 0.101 \cdot \phi'_d + \varepsilon_{E,d}) = 4097.65 \text{ kN/m}^2$$
$$\gamma_{sat,d} = \frac{(G_s + e_d)\gamma_w}{1+e_d} = 22.96 \text{ kN/m}^3$$
$$\gamma_{dry,d} = \frac{G_s \gamma_w}{1+e_d} = 20.93 \text{ kN/m}^3 \quad (17)$$

and the design values of other intermediate variables, such as $\gamma_{eff,d}$, $q_{eff,d}$, $\zeta_{\gamma,d}$, $\zeta_{q,d}$, …, $N_{\gamma,d}$, $N_{q,d}$, and $Q_{u(\text{cal}),d}$ (the subscript 'd' means design value). $Q_{u(\text{act}),d}$ can then be computed using Eq. (8):

$$Q_{u(\text{act}),d} = \exp\left[1.384 + 0.805\ln(Q_{u(\text{cal}),d}) + \varepsilon_{Q,d}\right] = 2183.12 \text{ kN} \quad (18)$$

Then, the design value of G (denoted by G_d) is

$$G_d = \exp\left[1.384 + 0.805\ln(Q_{u(\text{cal}),d}) + \varepsilon_{Q,d}\right] - DL_d - LL_d - W = 18.69 \text{ kN} \quad (19)$$

which is positive, meaning that B = 3 m does not exactly achieve G_d = 0. A smaller B is required for the next iteration. This design process is repeated until we find the exact B value that gives G_d = 0. This B value is found to be 2.98 m for this particular illustration example.

The actual reliability index, denoted by β_A, for the above design outcome can be readily calculated by Monte Carlo simulation (MCS) (sample size $n = 10^6$), because B now takes a known numerical value of 2.98 m. For this particular illustration example, the actual reliability index is equal to 2.97 (failure probability $p_f = 1.5 \times 10^{-3}$), which is reasonably close to the target reliability index β_T = 3.2 (target $p_f = 6.9 \times 10^{-4}$). For a more thorough verification, 1000 verification cases with $\rho = -0.5$ are randomly sampled from the rectangular volume formed by the ranges summarized in Table 2. These 1000 verification cases are independent of the 100 calibration cases. Each verification case goes through the same iterative design process described above to obtain the final design B value, and the actual reliability index β_A is obtained using MCS. Table 3 shows the statistics of the β_A values for these 1000 verification cases, including the average, COV, maximum value, and minimum value. If all 1000 β_A values are equal to the target value 3.2, QVM performs perfectly. For this perfect performance, the average = max = min = 3.2 and COV = 0. Conversely, a poor performance will be characterized by a large COV and a minimum value that is much less than 3.2. The COV for β_A is reasonably small (0.09), and the minimum value is deemed acceptable (2.53). The performance of QVM for ULS is deemed acceptable, because the actual reliability index for the resulting design outcome is reasonably close to the target value β_T.

2.4 Comparison with Eurocode 7

It is of interest to compare the performance of QVM to existing design codes. We consider Eurocode 7 herein. The partial factors recommended by Annex A in Eurocode 7 British Standard (British Standard Institute 2004) are adopted. The design value of a variable X, denoted by X_d, is equal to

$$X_d = \begin{cases} \gamma_X \cdot X_c & \text{if } X \text{ is destabilizing} \\ X_c / \gamma_X & \text{if } X \text{ is stabilizing} \end{cases} \quad (20)$$

Table 3 Statistics of the β_A values for the 1000 verification cases with $\rho = -0.5$

	Mean	COV	Maximum	Minimum
β_A	3.21	0.09	3.82	2.51

where γ_X is the partial factor for X, and X_c is the characteristic value for X. Table 4 shows the recommended partial factors for Design Approaches 1, 2, and 3. Note that Design Approach 3 has partial factors identical to Design Approach 1 Combination 2. Annex A does not recommend partial factors for void ratio (e). Because e is closely related to unit weight, the partial factor for unit weight is taken. The characteristic value X_c should be selected as a cautious estimate of the value affecting the occurrence of the limit state (Section 2.4.5.2 in British Standard Institute 2004). In the current example, X_c is taken to be the 0.05 quantile of X if X is stabilizing (ϕ') and 0.95 quantile if X is destabilizing (DL, LL, e). Note that this 0.05 quantile is not calibrated by reliability theory, but it is a prescribed number that is based on judgment.

Consider the same design example with μ_{DL} = 1000 kN, B/L = 1, D = 1 m, h = 0 m, G_s = 2.7, μ_e = 0.4, V_e = 0.2, $\mu_\phi = 35°$, V_ϕ = 0.1, $r_{L/D}$ = 0.5, and $\rho = -0.5$ and consider Design Approach 2. The design process is also iterative. Let us start with B = 3 m. The characteristic value for ϕ' is its 0.05 quantile: $\phi'_c = 29.56°$. The characteristic values for (DL, LL, e) are their 0.95 quantiles: DL_c = 1164.49 kN, LL_c = 686.58 kN, and e_c = 0.54. The design value of ϕ' is [note: $\gamma_{\tan(\phi')} = 1.0$]

$$\phi'_d = \tan^{-1}\left(\tan(\phi'_c)/\gamma_{\phi'}\right) = 29.56° \quad (21)$$

The design values for (DL, LL, e) are

$$DL_d = \gamma_{DL} \cdot DL_c = 1572.06 \text{ kN}$$
$$LL_d = \gamma_{LL} \cdot LL_c = 1029.87 \text{ kN}$$
$$e_d = \gamma_e \cdot e_c = 0.54 \quad (22)$$

Based on the design values (ϕ'_d, e_d, DL_d, LL_d), the design engineer can further calculate E_d, $\gamma_{sat,d}$, and $\gamma_{dry,d}$:

$$\text{E}_d = \exp(5.785 + 0.101 \cdot \phi'_d) = 11158.98 \text{ kN/m}^2$$
$$\gamma_{sat,d} = \frac{(G_s + e_d)\gamma_w}{1+e_d} = 20.62 \text{ kN/m}^3$$
$$\gamma_{dry,d} = \frac{G_s \gamma_w}{1+e_d} = 17.16 \text{ kN/m}^3 \quad (23)$$

and also $Q_{u(\text{cal}),d}$. $Q_{u(\text{act}),d}$ can then be computed using Eq. (8):

$$Q_{u(\text{act}),d} = \exp\left[1.384 + 0.805\ln(Q_{u(\text{cal}),d})\right] \big/ \gamma_{Q_u} = 2599.93 \text{ kN} \quad (24)$$

Note that there is an extra partial factor γ_{Q_u} in the denominator above. Then, the design value of G (denoted by G_d) is

Table 4 The recommended partial factors for Design Approaches 1, 2, and 3

Design Approach		γ_{DL}	γ_{LL}	$\gamma_{\tan(\phi')}$	γ_e	γ_{Q_u}
1	Combination 1	1.35	1.5	1.0	1.0	1.0
	Combination 2	1.0	1.3	1.25	1.0	1.0
2		1.35	1.5	1.0	1.0	1.4
3		1.0	1.3	1.25	1.0	1.0

$$G_d = \exp\left[1.384 + 0.805\ln(Q_{u(\text{cal}),d})\right]\Big/\gamma_{Q_u} - DL_d - LL_d - W = -227.00 \text{ kN} \tag{25}$$

which is negative, meaning that $B = 3$ m is insufficient. A larger B is required for the next iteration. This design process is repeated until we find the exact B value that gives $G_d = 0$. This B value is found to be 3.15 m for this particular illustration example. Again, 1000 verification cases are randomly sampled from the rectangular volume formed by the ranges summarized in Table 2. Each verification case goes through the same iterative design process to obtain the final design B value, and the actual reliability index β_A is obtained using MCS. Table 5 shows the statistics of the β_A values for these 1000 verification cases. The results for Design Approach 3 are not listed, because they are the same as those for Design Approach 1 Combination 2. It is clear that Design Approach 1 Combination 2 and Design Approach 2 perform similarly: the mean values of β_A ranged from 3.0 to 3.1. However, Design Approach 1 Combination 1 has a lower β_A mean value (2.32). The COV of β_A is small (0.05 ~ 0.07), indicating these partial factors together with the 0.05-quantile characteristic values can achieve designs with fairly uniform reliability indices.

The main difference between QVM and Eurocode 7 is that the η value in QVM is calibrated by reliability theory. The calibrated η value for QVM can be easily found from Fig. 2. Designs produced by QVM have actual reliability indices (β_A) that always centers at the target reliability index (β_T), *e.g.*, the mean value of β_A in Table 3 is very close to the target value 3.2. However, designs produced by Eurocode 7 have an unknown target reliability. The actual reliability of the resulting design depends on how the characteristic values are chosen. One can imagine the actual reliability index will decline if 0.5-quantiles (median) are adopted as the characteristic values rather than 0.05-quantiles.

3. SERVICEABILITY OF A FOOTING IN A COHESIONLESS SOIL

For the cases in the database collected by Akbas and Kulhawy (2009a, 2009b), the load-settlement curves for the footings are also known. Akbas and Kulhawy (2009a) developed the following hyperbolic equation to fit the load-settlement curve:

$$\frac{Q}{Q_{u(\text{act})}} = \left(\frac{S}{B}\text{ in }\%\right)\Big/\left[a\left(\frac{S}{B}\text{ in }\%\right) + b\right] \tag{26}$$

Table 5 Statistics of the β_A values for the 1000 verification cases with $\rho = -0.5$, based on the partial factors recommended by Annex *A* in Eurocode 7 British Standard

Design Approach		Mean	COV	Maximum	Minimum
1	Combination 1	2.32	0.07	2.63	1.68
	Combination 2	3.09	0.07	3.91	2.20
2		3.02	0.05	3.46	2.56

where Q is the loading; S is the settlement when the loading = Q; (a, b) are the two parameters for the hyperbolic model. Figure 3 shows the significance of the (a, b) parameters in the normalized load-settlement curve. For each case, the two parameters (a, b) are determined to best fit the observed normalized load-settlement curve. Figure 4 shows the values of the best-fit (a, b) parameters for all cases (dark crosses). The parameter a is roughly normally distributed with mean = 0.70 and variance = 0.50. The logarithm of parameter b, namely ln(b), is also roughly normally distributed with mean = 0.44 and variance = 4.34. There is a negative correlation between a and ln(b), and the correlation coefficient is −0.78. It is further assumed that a and ln(b) are jointly normal. The grey dots in Fig. 4 are the simulated (a, b) data points using the above jointly normal model. They behave similarly to the actual (a, b) data points. The inverse function of Eq. (26) has the following form:

$$\left(\frac{S}{B}\text{ in }\%\right) = \frac{b(Q/Q_{u(\text{act})})}{1 - a(Q/Q_{u(\text{act})})} \tag{27}$$

Based on the fact that $Q = DL + LL$,

$$S = (B/100)\cdot\frac{b(DL+LL)/Q_{u(\text{act})}}{1-\left[a(DL+LL)/Q_{u(\text{act})}\right]} \tag{28}$$

The performance function for the serviceability limit state (SLS) is

$$G = \ln(S_{allow}/S) \tag{29}$$

where S_{allow} is the allowable settlement for the footing foundation.

3.1 Random Variables

The basic random variables for the SLS performance function G include (ϕ', e, DL, LL, ε_Q, ε_E) (recall that $Q_{u(\text{act})}$ depends on ϕ', e, ε_E, and ε_Q). There are two extra random variables for SLS: (a, b). Table 1 already shows the assumed probability distributions for (ϕ', e, DL, LL, ε_Q, ε_E). a and ln(b) are assumed to follow the jointly normal distribution as mentioned above.

3.2 Calibration of the QVM

For SLS, the decision regarding the stabilizing/destabilizing random variables is the same as ULS: (ϕ', ε_Q, ε_E) are stabilizing, so they are reduced to their η quantiles to obtain the design values; (e, DL, LL) are destabilizing, so they are increased to their $1-\eta$ quantiles to obtain the design values. According to Fig. 3,

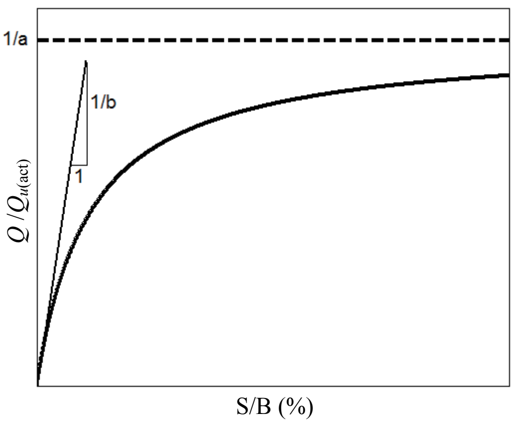


Fig. 3 The significance of the (*a*, *b*) parameters in the normalized load-settlement curve

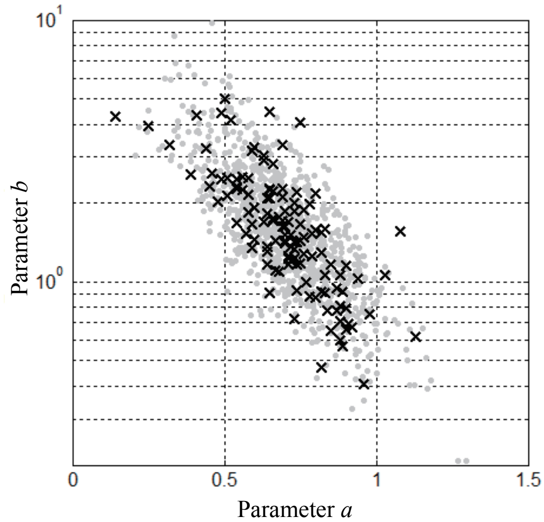


Fig. 4 The values of the best-fit (*a*, *b*) parameters for all cases (dark crosses); the grey dots are the simulated (*a*, *b*) data using the jointly normal model

the two extra random variables (a, b) should be destabilizing, because the increase of (a, b) will reduce the initial slope and asymptotic value of the load-settlement curve, hence increase the settlement. As a result, (a, b) are increased to their $1-\eta$ quantiles to obtain the design values.

There are twelve design parameters: (B, B/L, D, h, G_s, μ_e, V_e, μ_ϕ, V_ϕ, $r_{L/D}$, ρ) and S_{allow}. The ranges for (B, B/L, D, h, G_s, μ_e, V_e, μ_ϕ, V_ϕ, $r_{L/D}$, ρ) are already described in Table 2. The range of S_{allow} is taken to be among 10 mm, 25 mm, 50 mm, and 100 mm. A collection of 100 calibration cases are obtained from randomly sampling the rectangular volume formed by these ranges. These 100 calibration cases should cover a wide variety of future design cases because these ranges are very wide. Given a target reliability index (β_T), an η value is calibrated for each of the 100 calibration cases using the theory developed in Ching and Phoon (2011). The average of the 100 calibrated η values is then adopted in QVM. Figure 5 shows how the average η value changes with β_T under various S_{allow} values (this relationship is insensitive to ρ).

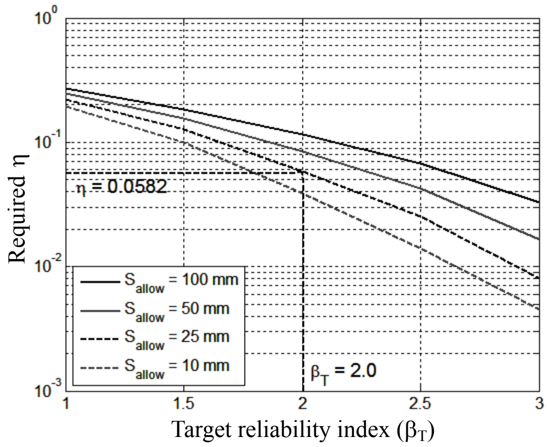


Fig. 5 The relationship between the average η value and β_T under various S_{allow} values (SLS)

3.3 Design Illustration

For illustration, the design scenario that was used previously (μ_{DL} = 1000 kN, B/L = 1, D = 1 m, h = 0 m, G_s = 2.7, μ_e = 0.4, V_e = 0.2, $\mu_\phi = 35^\circ$, V_ϕ = 0.1, $r_{L/D}$ = 0.5, $\rho = -0.5$) is considered here. Moreover, let S_{allow} be 25 mm. The goal is to design the width B such that the target reliability index (β_T) is equal to 2.0 for SLS, where the failure is defined as $\ln(S_{allow}/S) < 0$. The design process is iterative. Let us start with B = 3 m. For β_T = 2.0, the corresponding η value is equal to 0.0582, according to Fig. 5. The design values (ϕ'_d, $\varepsilon_{Q,d}$, $\varepsilon_{E,d}$, e_d, DL_d, LL_d, E_d, $\gamma_{sat,d}$, $\gamma_{dry,d}$) can be computed using Eqs. (12) ~ (17) with η = 0.0582. The resulting design values are $\phi'_d = 29.78^\circ$, $\varepsilon_{Q,d} = -0.46$, $\varepsilon_{E,d} = -0.80$, e_d = 0.29, DL_d = 1157.01 kN, LL_d = 674.41 kN, E_d = 5014.31 kN/m^2, $\gamma_{sat,d}$ kN/m^3 = 22.76, and $\gamma_{dry,d}$ = 20.57 kN/m^3. The design values of other intermediate variables, such as $\gamma_{eff,d}$, $q_{eff,d}$, $\zeta_{\gamma,d}$, $\zeta_{q,d}$, …, $N_{\gamma,d}$, $N_{q,d}$, and $Q_{u(\text{cal}),d}$, can be also computed. $Q_{u(\text{act}),d}$ can then be computed using Eq. (18). The resulting $Q_{u(\text{act}),d}$ is 2734.62 kN. (a, b) are increased to their $1-\eta$ quantiles to obtain the design values

$$a_d = 0.7 + \sqrt{0.5} \cdot \Phi^{-1}(1-\eta) = 0.94$$
$$b_d = \exp\left[0.44 + \sqrt{4.34} \cdot \Phi^{-1}(1-\eta)\right] = 1.44 \tag{30}$$

As a result, the design value of the settlement S (denoted by S_d) is

$$S_d = \frac{B}{100} \cdot \frac{b_d (DL_d + LL_d) / Q_{u(\text{act}),d}}{\left\{1 - \left[a_d (DL_d + LL_d) / Q_{u(\text{act}),d}\right]\right\}}$$
$$= 0.0785 \text{ m} = 78.5 \text{ mm} \tag{31}$$

Then, the design value of G (denoted by G_d) is

$$G_d = \ln(S_{allow} / S_d) = -0.1443 \tag{32}$$

which is negative, indicating that B = 3 m is insufficient. A larger B is required for the next iteration. This design process is repeated until we find the exact B value that gives G_d = 0. This B value is found to be 4.79 m for this particular illustration example.

The actual reliability index, denoted by β_A, for the above design outcome can be readily obtained by Monte Carlo simulation

(MCS) (sample size $n = 10^6$), because B now takes a known numerical value of 4.79 m. In MCS, the failure is defined as $\ln(S_{allow}/S) < 0$. For this particular illustration example, the actual reliability index is equal to 1.76, which is reasonably close to the target reliability index $\beta_T = 2.0$. For a more thorough verification, 1000 verification cases with $\rho = -0.5$ and $S_{allow} = 25$ mm are randomly sampled from the rectangular volume formed by the ranges summarized in Table 2. These 1000 verification cases are independent of the 100 calibration cases. Each verification case goes through the same iterative design process described above to obtain the final design B value, and the actual reliability index β_A is obtained using MCS. Table 6 shows the statistics of the β_A values for these 1000 verification cases, including the average, COV, maximum value, and minimum value. The COV for β_A is acceptable (0.15), and the minimum value is deemed acceptable (1.56). The performance of QVM for SLS is deemed acceptable, because the actual reliability index of the resulting B is reasonably close to the target value β_T. Comparison to designs based on Eurocode 7 is not taken here, because the guideline for SLS is less clear in Eurocode 7 than the guideline for ULS.

Table 6 Statistics of the β_A values for 1000 verification cases with $\rho = -0.5$ and $S_{allow} = 25$ mm

	Mean	COV	Maximum	Minimum
β_A	2.04	0.15	3.24	1.56

4. CONCLUSIONS

In this paper, the quantile value method (QVM) is used to calibrate a simplified reliability-based design (RBD) code. The subject of interest is limited to the capacities of footings in cohesionless soils. Both ultimate limit state (ULS) and serviceability limit state (SLS) are considered. The calibration takes advantage of a footing database recently collected by Akbas and Kulhawy (2009a, 2009b). The data points in this database are used to develop the probability distribution of the modeling errors. It is correct to say that the calibrated QVM codes already incorporate these modeling errors.

For both ULS and SLS, design charts are developed based on the QVM calibration. Design engineers can easily find the required η value based on the target reliability index (β_T). The design value for each random variable can be taken to be its η-quantile if it is a stabilizing variable (*e.g.*, soil strength) or to be its $(1-\eta)$-quantile if it is a destabilizing variable (*e.g.*, load). For ULS, the η versus β_T relationship depends on the correlation coefficient between friction angle and void ratio of the cohesionless soil. For SLS, this relationship depends on the allowable settlement. It is shown that the designs based on the design charts can effectively achieve the target reliability index. However, readers are to be alerted that the design charts will be no longer applicable if any one of the following scenarios happens:

1. A Q_u calculation model different from Eqs. (1) ~ (5) [or Eq. (26)] is adopted.
2. The distribution types of the basic random variables are not the same as those assumed in Table 1.
3. The design parameters are outside the ranges summarized in Table 2.

The partial factors recommended by Annex A in Eurocode 7 are also implemented to the ULS design. It is shown that these partial factors can achieve a fairly uniform reliability index. However, the target reliability is unknown, because it depends on how the design engineer chooses the characteristic value. In this aspect, QVM is superior to these partial factors because QVM can effectively achieve a prescribed target reliability.

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