TGS Geotechnical Lecture:

ROBUST GEOTECHNICAL DESIGN — METHODOLOGY AND APPLICATIONS

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ABSTRACT

In this paper, the authors present the concept of robust geotechnical design, a new design methodology used to make the response of a geotechnical system insensitive to, or robust against, the variation of uncertain geotechnical parameters (called noise factors herein). By carefully adjusting the design parameters (those that can be controlled by the designer), this methodology is realized through a multi-objective optimization, in which all the design requirements such as safety, robustness, and cost are explicitly considered. The results of such optimization are often expressed as a Pareto Front, a collection of optimal designs that collectively define a trade-off relationship between cost and robustness, whereas the safety requirement is met. The Pareto Front enables the engineer to make an informed design decision according to a target cost or robustness. The significance and versatility of new design methodology are illustrated with several geotechnical applications including design of shallow foundations, drilled shafts and braced excavations.

Key words: Uncertainty, reliability, robust design, optimization, foundation, braced excavation.

1. INTRODUCTION

This paper presents the new robust geotechnical design (RGD) concept, a design methodology designed to achieve a certain level of design robustness, in addition to meeting the safety and cost requirements. The robust design, originated from the field of Industry Engineering (Taguchi 1986; Chen *et al.* 1996), has recently been applied to many design fields such as mechanical, structural and aeronautical design (*e.g.*, Doltsinis *et al.* 2005; Zhang *et al.* 2005; Park *et al.* 2006; Brik *et al.* 2007; Lagaros and Fragiadakis 2007; Kumar *et al.* 2008; Marano *et al.* 2008; Lee *et al.* 2010; Paiva 2010). This robust design concept is adapted herein to formulate a novel RGD methodology. Here, a design is considered "robust" if the variation in the system response is insensitive to the variation of noise factors (mainly uncertain geotechnical properties) and correlations between these noise factors.

In routine geotechnical practice, engineers are often limited in their ability to ascertain geotechnical parameters because of the complexity of soil deposits and the limited availability of soil data (due to the budget constraint). Furthermore, the uncertainty of geotechnical parameters can come from many sources, such as spatial variability, measurement errors, and transformation errors (Phoon and Kuhalwy 1999). To compensate for these uncertainties, the engineer usually chooses to adopt a conservative design that can be very cost inefficient. Today, the reliability-based design (RBD) approach is often a method of choice to deal with parameter uncertainties in the design (*e.g.*, Harr 1987; Christian *et al.* 1994; Wu *et al.* 1989; Duncan 2000; Baecher and Christian 2003; Phoon *et al.* 2003; Chalermyanont and Benson 2004; Fenton and Griffiths 2008; Hsiao *et al.* 2008; Najjar and Gilbert 2009; Griffiths and Fenton 2009; Ching and Phoon 2011; Juang *et al.* 2011; Wang *et al.* 2011; Zhang *et al.* 2011). However, because the uncertainties of geotechnical parameters are often difficult to characterize statistically, the RBD is often conducted with the assumed statistics of these uncertain parameters. As shown in Juang *et al.* (2013a), the results of RBD can be greatly affected by the assumed statistics (such as the coefficient of variation) of the geotechnical parameters. Thus, the dilemma of over-design for safety or under-design for cost efficiency has not been fully overcome even the design approach has evolved from the factor of safety (F_S)-based methods to the RBD methods.

The RGD approach advocated in this paper is an attempt to ease the dilemma of the current design methods (either F_S-based or RBD methods). This new design approach attempts to minimize the effects of uncertainties in the estimated statistics of geotechnical parameters by carefully adjusting the design parameters of the geotechnical system so that the predicted response of the system is insensitive to these uncertainties. The RGD approach is realized through multi-objective optimization that explicitly considers safety, cost, and robustness. Through this multi-objective optimization, a Pareto Front is established, which consists of a set of optimal designs and collectively defines some trade-off relationship. For example, when the safety requirement is met, the Pareto Front defines a trade-off between cost and design robustness. Thus, the Pareto Front can render the most optimal design if the desired cost range or the target robustness level is specified.

In this paper, the RGD approach is demonstrated with three design examples, including shallow foundations, drilled shafts, and braced excavations. The significance and versatility of the RGD approach is presented.

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2. ROBUST DESIGN CONCEPT

As noted previously, the robust design concept originated in the field of industry engineering, which we adapted in our study of geotechnical problems. A brief introduction of the robust design concept with a geotechnical perspective is presented below based on the prior work by the authors (Juang *et al*. 2013a).

In a traditional geotechnical design process, multiple candidate designs are first checked against safety requirements, and the acceptable designs are then optimized for cost, which yields the final design. In this design process, the safety requirements are analyzed by either deterministic methods or probabilistic methods. The deterministic methods use factor of safety (F_S) as a measure of safety, while probabilistic methods use reliability index or probability of failure as the measure of safety. With the F_S-based approach, the uncertainties in soil parameters and the associated analysis model are not considered explicitly in the analysis but their effect is considered in the design by adopting a threshold F_S value. With the probabilistic approach (or reliability-based design), these uncertainties are included explicitly in the analysis and the safety requirement is considered met if the reliability index or failure probability satisfies a threshold value. Finally, the cost optimization among the acceptable designs is performed to yield the final design.

Regardless of whether the F_S-based approach or the reliability-based approach is employed, the traditional design focuses mainly on safety and cost; design "robustness" is not explicitly considered. Robust design aims to make the product of a design insensitive to (or robust against) "hard-to-control" input parameters (called "noise factors") by adjusting "easy-to-control" input parameters (called "design parameters"). The essence of this design approach is to consider robustness explicitly in the design process along with safety and cost requirements.

There are two main drawbacks to traditional design approaches that fail to consider the robustness against noise factors (such as soil parameters variability and/or construction noise). First, the lowest-cost design may no longer satisfy the safety requirements if the actual variations of the noise factors are underestimated. Thus, the safety requirements can easily be violated because of the high variation of the system response due to the unexpectedly higher variation of noise factors. Second, the potentially high variability of the system response may force the designer to select an overly conservative design that guarantees safety, thus resulting in an inefficient and costly design. This dilemma between the over-design for safety and the under-design for cost-savings is, of course, an old problem in geotechnical engineering. However, by reducing the variation of the system response to ensure the design robustness against noise factors, the RGD approach can ease these decision-making dilemmas. Of course, the variation of the system response can also be reduced by reducing the variation in soil parameters. However, in many geotechnical projects the ability to reduce soil variability is restricted by the nature of the soil deposit (*i.e.*, possibility of inherent soil variability) and/or of the amount of soil test data that can be obtained. This proposed RGD methodology seeks a reduction in the variation of system response by adjusting only the design parameters (such as dimension and geometry), and not the noise factors (assuming that a reasonable site investigation has been performed).

It should be noted that using the concept of robust design to adjust the design parameters is just one option for meeting the design requirements. It may also be possible to meet design requirements by improving the soil parameter characterization. A balanced approach entails adopting a suitable site characterization and testing program, followed by a robust design with the estimated parameter uncertainty.

It must be emphasized that RGD is not a design methodology to replace the traditional F_S-based and reliability-based approaches; rather, it is a strategy used to complement these traditional methods. With the RGD approach, the focus involves satisfying three design objectives: safety, cost, and robustness (against the variation in system response caused by noise factors). The RGD approach is implemented in this paper as an optimization problem. As with many multi-objective optimization problems, it is possible that no single best solution exists that satisfies all three objectives. In such situations, a detailed study of the trade-offs among these design objectives may lead to a more informed design decision.

In this paper, robustness is first considered within the framework of a reliability-based design. Specifically, a reliability-based RGD procedure is proposed herein and illustrated with design examples of shallow foundations and drilled shafts. A slight variation of the RGD procedure is then presented for the design of braced excavation using allowable wall deflection and factors of safety, in lieu of reliability indexes, as a constraint. In the sections that follow, a brief introduction of robust geotechnical design (RGD) methodology is presented, followed by the illustrative examples demonstrating the significance of the design robustness and the effectiveness of this methodology for selection of the "best" design based upon multiple objectives.

3. ROBUST DESIGN METHODOLOGY

An outline for robust geotechnical design (RGD) approach is presented below. In reference to Fig. 1, the RGD approach is summarized in the following steps (Juang *et al*. 2012; Juang *et al*. 2013a & b; Wang *et al.* 2013):

Step 1: Define the problem of concern and classify all input parameters of the intended geotechnical system into the design parameters and the noise factors. For the given problem, the deterministic model (limit state or performance function) of the intended geotechnical system is then established.

Step 2: Determine the statistics of the uncertain geotechnical parameters and characterize the uncertainty in the statistics of these noise factors and identify the design domain. For the design of geotechnical systems, the key uncertain soil parameters are usually identified as noise factors. The uncertainty in the statistics (*e.g.*, coefficient of variation) of each of the noise factors may be estimated based on published literatures guided by engineering judgment or the bootstrapping method based on limited data. For the design parameters, the design domain should be defined based upon their typical ranges, augmented with local experiences. These design parameters should be specified in discrete numbers for convenience in construction. Thus, the design domain will consist of a finite number (M) of designs.

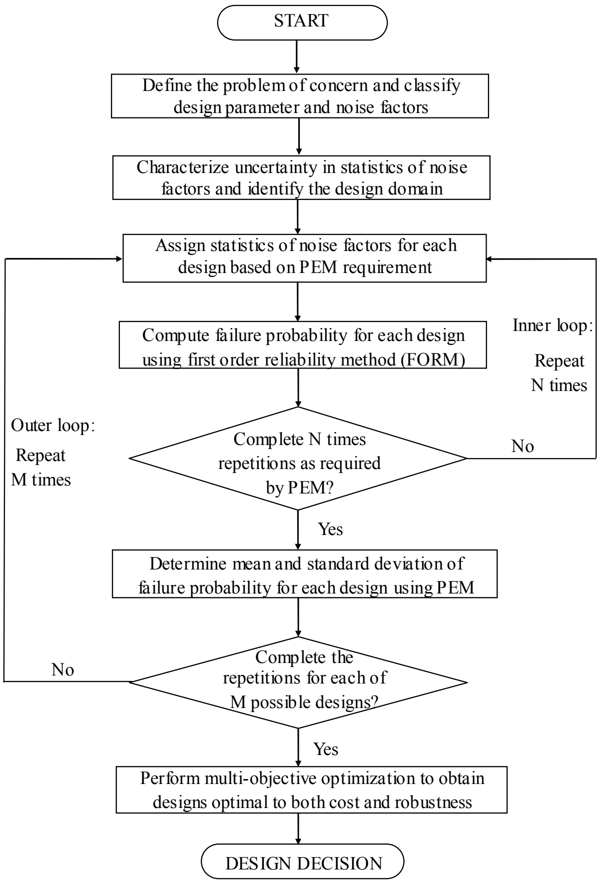

Fig. 1 Flowchart for Robust Geotechnical Design (modified after Juang *et al.* 2013a)

Step 3: Evaluate the variation of the system response as a measure of robustness of a given design. In this paper, the critical failure probability based on either ultimate limit state (ULS) or serviceability limit state (SLS), or the critical deformation that controls the design, is used as a measure of system response. Recall that a design is considered robust if the variation of its system response (*e.g.*, failure probability or deformation) caused by the uncertainty in variation of noise factors is small. The variation of the system response is mainly caused by the variation of the derived statistics of the noise factors. Thus, in this step the mean and standard deviation (as a measure of robustness) of the system response will be evaluated. In this paper, the point estimate method (PEM) as updated by Zhao and Ono (2000) is adopted for such evaluations.

In the flowchart shown in Fig. 1, the failure probability of the system is treated as the system response. A slight modification is needed when critical deformation is adopted as the system response. In either case, PEM can be adopted for evaluation of the variation in the system response.

The PEM approach requires evaluation of the system response (*e.g.*, failure probability) at each of a set of N "estimating" points of the input noise factors, as reflected by the inner loop shown in Fig. 1 (Juang *et al*. 2013a). In each repetition, the statistics of each of the noise factors at each PEM estimating point must be assigned, so that the system response (using failure probability as an example) can be computed using the first order reliability method (FORM; Ang and Tang 1984). The resulting N failure probabilities are then used to compute the mean and standard deviation of the failure probability (the system response).

Step 4: Conduct the repetitive analysis in Step 3 for each possible design in the design domain. For each design, the mean and standard deviation of the failure probability (the system response) are determined. This step is represented by the outer loop shown in Fig. 1.

Step 5: Perform a multi-objective optimization to establish a Pareto Front, which serves as a guide for choosing the most preferred design.

Note that the geotechnical design is indeed a design problem with multiple criteria, including safety, cost and robustness. In the actual implementation, the mean failure probability obtained from PEM integrated with FORM is set as the safety constraint to screen the unsatisfactory designs, and cost (in terms of construction cost for the geotechnical system) and robustness (in terms of standard deviation of the failure probability) are set as two objectives for multi-objective optimization.

The Pareto Front concept is briefly introduced in Fig. 2. When multiple conflicting objectives are enforced, it is likely that no single best design exists that is superior to all other designs in all objectives. However, a set of designs may exist that are superior to all other designs in all objectives; within the set, however, none is superior or inferior to others in all objectives (Juang *et al*. 2012). This set of optimal designs constitutes a Pareto Front (Ghosh and Dehuri 2004).

In this paper, the Pareto Front is established through the Non-dominated Sorting Genetic Algorithm version II (NSGA-II), developed by Deb *et al*. (2002). The obtained Pareto Front can be used for selecting the most preferred design if the desired cost/robustness level is specified.

4. RGD APPLICATION: EXAMPLE NO. I – DESIGN OF DRILLED SHAFT

The example presented in this section is a summary of the prior work by the authors (Juang *et al*. 2013a), and the reader is referred to that work for additional details.

4.1 Illustrative Example

A design example of the drilled shaft in loose sand, subjected to an axial load under drained conditions (Juang *et al*. 2013a), as shown in Fig. 3, is first used to demonstrate the application of the proposed RGD approach. For the loose sand in this example, the total unit weight is 20 kN/m^3, the mean value of the effective friction angle φ' is 32° and the mean value of the coefficient of earth pressure at rest K_0 is 1. The unit weight of concrete is 24 kN/m^3 and the nominal operative *in situ* horizontal stress coefficient ratio is 1.0. The water table is set at the ground surface. The design parameters for the shaft are the diameter B and depth (length) D, respectively.

The drilled shaft is designed to satisfy the requirements of both ultimate limit state (ULS) and serviceability limit state (SLS). For either ULS or SLS requirement, the drilled shaft is considered failed if the compression load exceeds the shaft compression capacities. In this study, the axial compression load F is

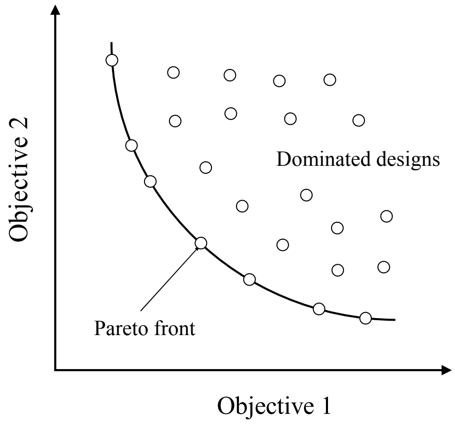

Fig. 2 Illustration of Pareto Front (modified after Juang *et al.* 2013b)

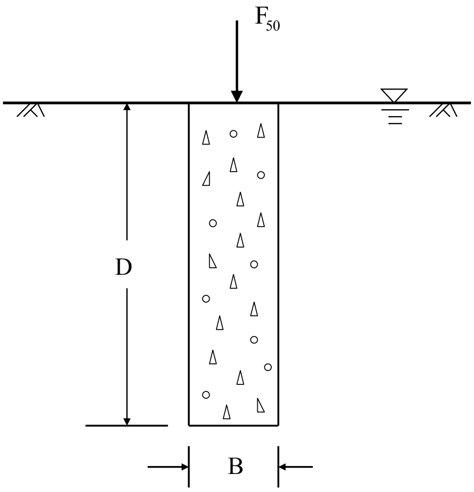

Fig. 3 Schematic illustration of drilled shaft design example (modified after Phoon *et al.* 1995)

set as the 50-year return period load F_{50} (F_{50} = 800 kN in this example) with an allowable settlement of 25 mm. Here, the ULS compression load capacity (denoted as Q_{ULS}) is determined using the procedure developed by Kulhawy (1991), and the SLS compression capacity (denoted as Q_{SLS}) is determined using the method of normalized load-displacement curve for drilled shaft developed by Phoon *et al*. (1995).

4.2 Uncertainty Modeling

The two uncertain soil parameters in the design of a drilled shaft in loose sands, as pointed out by Phoon *et al*. (1995), are the drained friction angle φ' and the coefficient of earth pressure at rest K_0. As noted previously, these two uncertain parameters are treated as noise factors within the RGD framework.

In geotechnical practice, soil parameters are often determined from a small set of test data; thus, the statistical characterization based on a small sample may be subjected to error. In general, the "population" mean can be adequately estimated from the "sample" mean even with a small sample (Wu *et al*. 1989). However, the estimation of variation of the population based on a small sample is often not as accurate. The variation (in terms of the coefficient of variation, COV) is usually estimated based on the typical ranges reported. In this paper, the published COVs are adopted to illustrate the robustness concept in a geotechnical design. The COV of φ' of loose sand, denoted as COV [φ'] , typically ranges from 0.05 to 0.10 (Amundaray 1994), and the COV of K_0, denoted as COV [K_0], typically ranges from 0.20 to 0.90 (Phoon *et al*. 1995). Based on these typical ranges, the mean of COV [φ'] is assumed as 0.07, and the coefficient of variation of COV [φ'] is assumed as 18%. Similarly, the mean of COV [K_0] is assumed as 0.5 and its coefficient of variation of COV [K_0] is assumed as 30%. Furthermore, φ' and K_0 of loose sand are known negatively correlated. Based on the typical ranges for the correlation coefficient between φ' and K_0 (denoted as $\rho_{\varphi', K0}$). The mean of $\rho_{\varphi', K0}$ is assumed as −0.75 and its coefficient of variation of COV [K_0] is assumed as 10%. Though these assumed values are used to illustrate the design example here, in the real world, these values are estimated by the engineer. Nevertheless, the required level of precision of such estimates is not high, and a range estimate based on reported ranges in the literature, augmented with local experience will suffice. Thus, it should not be an extra burden to the engineer who is knowledgeable in the traditional RBD approach.

4.3 RGD of Drilled Shaft

As noted previously, the diameter *B* and depth *D* of drilled shaft represent the design parameters, the design domain of which should be specified. The choice of diameter *B* is usually limited to equipment and local practice, and for illustration purposes in this paper, only three discrete values (B = 0.9 m, 1.2 m, and 1.5 m) are considered here. The depth *D* typically ranges from 2 m to 8 m with an increment of 0.2 m (Wang *et al*. 2011). Thus, design parameters *B* and *D* can be conveniently modeled in the discrete domain with finite number of designs (say, *M* designs, M = 93 in this example).

For the safety requirements, the target failure probability for SLS is set as 0.0047(corresponding to a reliability index of 2.6) and the target failure probability for ULS is set as 0.00069 (corresponding to a reliability index of 3.2). A sensitivity study indicates that the SLS requirement controls the design of drilled shaft under axial compression load, which is consistent with those reported by other investigators (*e.g.*, Wang *et al*. 2011). Indeed, in all analyses performed in this study, the SLS requirement always controls the design of drilled shafts in sand for axial compression. Thus, in the subsequent analysis only the SLS failure probability is considered.

Following the RGD procedure described previously, PEM integrated with FORM procedure is used to compute the mean and standard deviation of the SLS failure probability for each design in the design domain. The cost of each design is computed using the cost estimate method proposed by Wang *et al*. (2011). The non-dominant sorting technique is then used to select the non-dominated optimal designs, which are points on the Pareto Front optimal to both cost and robustness. For the geotechnical design of drilled shafts in sand, the goal of the multi-objective optimization is to maximize the design robustness (or minimizing the standard deviation of the SLS failure probability) and to minimize the cost for constructing the drilled shaft, while ensuring that the safety constraint is satisfied (in this paper, the mean of the SLS failure probability is less than the target failure probability), as shown in Fig. 4.

Find	**d** = [B, D]
Subject to:	$B \in$ {0.9m, 1.2m, 1.5m}
	$D \in$ {2m, 2.2m, 2.4m, … , 8m}
	$\mu_p^{SLS} < p_T^{SLS} = 0.0047$
Objectives:	Minimizing the std. dev. of SLS failure probability
	Minimizing the cost for drilled shaft

Fig. 4 Formulation of multi-objective optimization for design of drilled shaft

Through multi-objective optimization, a converged Pareto Front is obtained, which consists of 27 designs that satisfy the safety constraint and are optimal to both objectives of cost and robustness. This Pareto Front is shown in Fig. 5. A trade-off relationship between two objectives is implied. Though the least cost design on the Pareto Front can be identified with B = 0.9 m and D = 6.0 m, it is the least robust. The trade-off relationship as shown in Fig. 5 enables an informed decision to be made when a desired cost level or a robustness level is specified. When the desired cost level is specified (using Fig. 5 as a guide), the design with the highest robustness is the best design; and when the desired robustness level is specified (using Fig. 5 as a guide), the design with the least cost at the desired robustness level is the best design.

Although the Pareto Front provides a trade-off relationship that aids in informed decision-making, it may be desirable to use a more user-friendly index to measure robustness. The feasibility robustness is defined as the confidence probability that the actual failure probability satisfies the target failure probability in the face of uncertainty, which is expressed as follows (Juang *et al.* 2013a):

$$Pr\left[(p_f - p_T) < 0\right] = Pr\left[(\beta_T - \beta) > 0\right] = \Phi\,(\beta_\beta) > P_0 \qquad (1)$$

where p_f is the computed failure probability, which is a random variable affected by uncertainty in the estimated statistics of noise factors; p_T is the target failure probability; $Pr\ [(p_f - p_T) < 0]$ is the confidence probability that the target failure probability requirement is satisfied; and P_0 is an acceptable level of the confidence probability selected by the designer. $Pr[(p_f - p_T) < 0]$ is equivalent to $Pr\ [(\beta_T - \beta) > 0]$, which can be computed using the first-order second-moment method. Thus, the feasibility robustness index β_β can be used as a measure for feasibility robustness, which corresponds to different confidence probability P_0 that the failure probability requirement is satisfied under the uncertainty in statistics of noise factors.

The β_β value for each design on the Pareto Front is then computed and the relationship between β_β and corresponding cost is shown in Fig. 6. With Fig. 6, by selecting a desired feasibility robustness level (in terms of β_β), the least-cost design among those on the Pareto Front can readily be determined as shown in Table 1. For example, when the feasibility robustness level is set at β_β = 1, which corresponds a confidence probability of 84.13%, then the least cost design on the Pareto Front satisfies the feasibility robustness requirement can be easily identified with B = 0.9 m and D = 6.2 m and a cost of 1,602 USD. Thus, the feasibility robustness offers an ease-to-use measure for making a more informed decision.

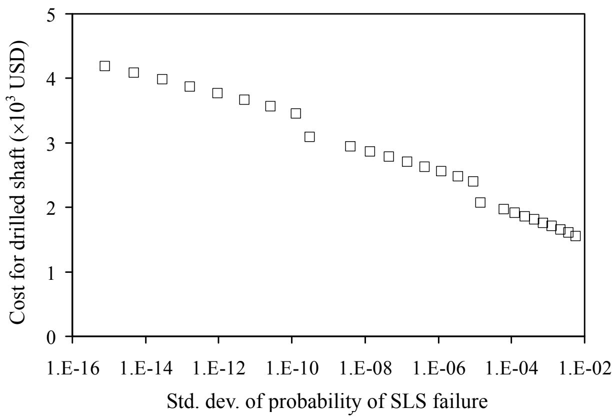

Fig. 5 Pareto Front for design of drilled shaft

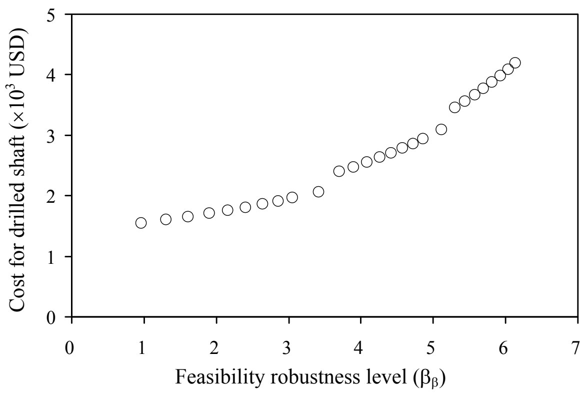

Fig. 6 Cost versus feasibility robustness for design of drilled shaft

Table 1 Final designs at selected feasibility robustness levels for drilled shaft (data from Juang *et al.* 2013a)

β_β	P_0	B (m)	D (m)
1	84.13%	0.9	6.2
2	97.72%	0.9	6.8
3	99.87%	0.9	7.6

5. RGD APPLICATION: EXAMPLE NO. II – DESIGN OF SHALLOW FOUNDATION

The example presented in this section is a summary of the prior work by the authors (Juang *et al.* 2012), and the reader is referred to that work for additional details.

5.1 Illustrative Example

A design example of the shallow foundation is used to further demonstrate the proposed RGD approach, as shown in Fig. 7. A square foundation ($B = L$) is used to support vertical compressive loads with a permanent load component of $G = 2000$ kN and a transient load component of $Q = 1000$ kN. G and Q are assumed to follow lognormal distribution with a COV of G of 10% and a COV of Q of 18% (Zhang *et al*. 2011). The soil profile at the site is assumed to follow the example presented by Orr and Farrel (1999), which consists of homogeneous dry sand with a deterministic unit weight of 18.5 kN/m^3. Ten effective friction angles φ' (for dry sand, $c' = 0$) are obtained from triaxial tests conducted on samples of this homogeneous sand, which are listed in Juang *et al*. (2012). The groundwater table is assumed as very deep such that it has negligible effects on the foundation design. The maximum allowable settlement is set at 25 mm.

The ULS capacity of shallow foundation is determined using Vesić model (Vesić 1975) updated by Kulhawy *et al*. (1983). The SLS capacity of shallow foundation is determined using normalized load-settlement method developed by Akbas and Kulhawy (2009a; 2009b). The foundation failure is said to occur if the ULS or SLS bearing capacity is less than the applied load (combination of permanent load G and transient load Q).

5.2 Uncertainty Modeling

For the design of a shallow foundation in cohesionless soils, soil parameter φ', the ULS model bias factor BF, and the two curve fitting parameters a and b of the SLS model are identified as noise factors. The uncertainty in the statistics of each of these noise factors is estimated with a bootstrapping method (Luo *et al*. 2012). With the ten effective friction angles φ', the variation in the mean and standard deviation of φ' can be estimated using bootstrapping method. The results show that the variation of sample mean is quite negligible (COV of sample mean is 1.7%), while the variation of sample standard deviation is large (COV of sample standard deviation is 17.9%). Similarly, the bootstrapping method is also used to estimate the variation in sample statistics of other noise factors (*e.g.*, BF, a and b).

5.3 RGD of Shallow Foundation

In the geotechnical design of a square shallow foundation, the design parameters are the foundation width B and the embedment depth D. The footing width B typically ranges from a minimum value of 1 m to a maximum value of 5 m (Akbas and Kulhawy 2011). The foundation embedment depth D typically ranges from 1 m to 2 m (Wang and Kulhawy 2008). For convenience of construction, the foundation dimensions are rounded to the nearest 0.1 m (Wang 2011). Thus, there are a finite number of designs in the design domain; in this paper, the number of designs is $M \approx 450$.

For the safety requirements, the reliability requirements defined in Eurocode 7 are adopted for this foundation design. The target failure probability for ULS is set as 0.000072 (corresponding to a reliability index of 3.8) and the target failure probability for SLS is set as 0.067 (corresponding to a reliability index of 1.5). A sensitivity study indicates that the ULS requirement controls the design of shallow foundation under vertical compression load, which is consistent with those reported by other investigators (*e.g.*, Wang 2011). Indeed, in all analyses performed in this study, the ULS requirement always controls the design of shallow foundation for vertical compression. Thus, only the ULS failure probability is considered in the subsequent analysis.

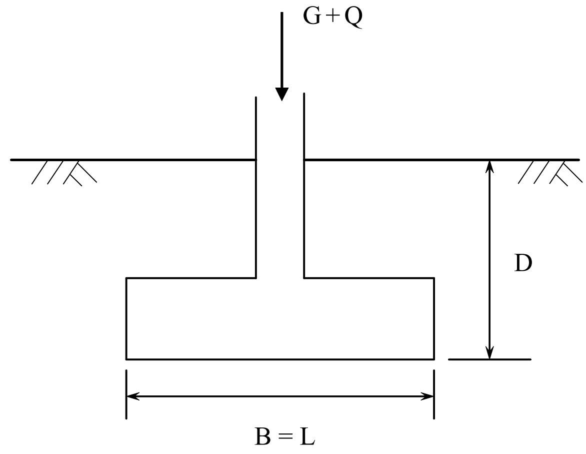

Fig. 7 Schematic illustration of shallow foundation design example (modified after Juang *et al.* 2012)

Following the flowchart of the RGD methodology presented in Fig. 1, the mean and standard deviation of the ULS failure probability can be obtained for all designs in the design domain using PEM integrated with FORM procedure. The cost for each design in the design domain can be calculated using the cost estimation procedure proposed by Wang and Kulhawy (2008). Then the multi-objective optimization using NSGA-II may be achieved by treating the target failure probability as a constraint and the robustness and cost as objectives, as shown in Fig. 8.

The design parameters (B and D in this case) are generated in the discrete space and all possible designs identified. Optimization is performed using NSGA-II. For this shallow foundation design, 62 "unique" designs are selected into the converged Pareto Front, as shown in Fig. 9. Note the obvious trade-off relationship between cost and robustness. The obtained Pareto Front can be used as a design aid for the decision maker to select the "best" design based on the desired target cost or robustness level, as every design on the Pareto Front meets the safety requirements.

Using the drilled shaft robust design procedure discussed in the previous section, the feasibility robustness index β_β for each of 62 designs on the Pareto Front of Fig. 9 can be computed, the results of which are shown in Fig. 10. As expected, a design with higher feasibility robustness (higher β_β) requires a higher cost. By selecting a target feasibility robustness level, the least-cost design among all on the Pareto Front can readily be identified as shown in Table 2. For example, when the feasibility robustness level is set at $\beta_\beta = 1$, which corresponds to a confidence probability of 84.13%, the least-cost design is $B = 2.1$ m and $D = 1.9$ m, which costs 1,200 USD. The feasibility robustness Pareto Front offers an easy-to-use measure for making an informed decision considering cost and robustness after satisfying the safety requirements.

Find $\mathbf{d} = [B, D]$

Subject to: $B \in \{1.0\text{m}, 1.1\text{m}, 1.2\text{m}, \ldots, 5.0\text{m}\}$

$D \in \{1.0\text{m}, 1.1\text{m}, 1.2\text{m}, \ldots, 2.0\text{m}\}$

$\mu_p^{ULS} < p_T^{ULS} = 0.000072$

Objectives: Minimizing the std. dev. of ULS failure probability

Minimizing the cost for shallow foundation

Fig. 8 Formulation of multi-objective optimization for design of shallow foundation

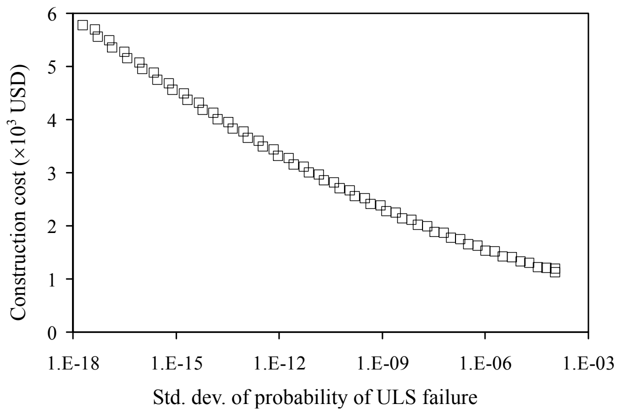

Fig. 9 Pareto Front for design of shallow foundation

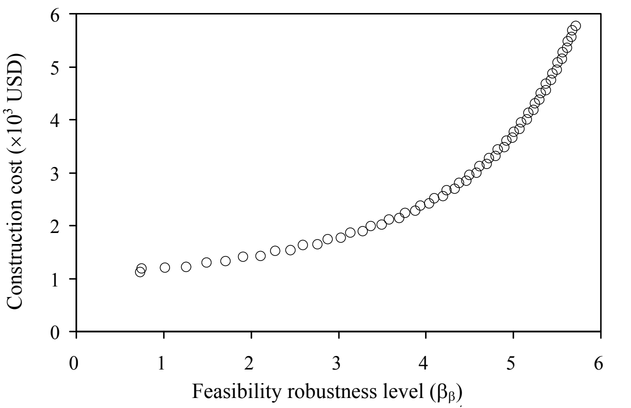

Fig. 10 Cost versus feasibility robustness for design of shallow foundation

Table 2 Final designs at selected feasibility robustness levels for shallow foundation (data from Juang *et al.* 2012)

β_β	P_0	B (m)	D (m)
1	84.13%	2.1	1.9
2	97.72%	2.3	2.0
3	99.87%	2.6	2.0

6. RGD APPLICATION: EXAMPLE NO. III – BRACED EXCAVATION DESIGN

The example presented in this section is a summary of the prior work by the authors (Juang *et al*. 2013b). The reader is referred to that research for additional details.

6.1 Illustrative Example

This final example concerns the design of braced excavation in clays. The soil profile at the excavation site consists of a homogenous clay layer with the ground water table located at 2 m below the ground surface. The clay is assigned a deterministic unit weight of 1.9 ton/m^3. The excavation site is rectangular in shape with a length of 40 m and a width of 25 m. The final excavation depth is 10 m and the diaphragm wall with multiple struts was employed as the retaining structure for the excavation. Figure 11 shows a schematic illustration of the braced excavation example with the vertical spacing of the struts $S = 2$ m.

The design of braced excavation in clays must satisfy the stability and deformation requirements. The stability requirements are enforced to prevent wall and ground failures (including push-in and basal heave failure for stability of excavation in clays). Factors of safety against these failures specified in the design code are enforced in the design. On the other hand, the deformation requirements are usually enforced to prevent damage to adjacent structures. In practice, the maximum wall deflection during the excavation is most often used as a measure for field control. Indeed, stability and deformation problems can generally be prevented when the maximum wall deflection is kept below a threshold value. Thus, the maximum wall deflection is herein considered as the response of concern for a braced excavation system, and a design is deemed robust if the variation of the maximum wall deflection caused by the uncertain noise factors (including soil parameters and construction noise) are small. In this study, a computer code TORSA (Taiwan Originated Retaining Structure Analysis) created by Trinity Foundation Engineering Consultants Co. (TFEC), which is a popular design tool based upon the beam-on-elastic foundation theory, was adopted as the deterministic model for prediction of the maximum wall deflection.

6.2 Uncertainty Modeling

For the design of braced excavation in clays, the uncertain soil parameters and construction noise are considered as noise factors. As an example, let us consider a braced excavation in a typical clay site in Taipei, where the normalized undrained shear strength (s_u / σ'_v) typically has a mean of 0.32 and a COV of 0.2, and the normalized modulus of horizontal subgrade reaction (k_h / σ'_v) typically has a mean of 48 and a COV of 0.5. These two soil parameters are generally correlated, and the correlation coefficient is estimated at approximately 0.7. The construction noise mainly refers to the surcharge behind the wall q_s, which is assumed to have a mean of 1 ton/m and a COV of 0.2.

6.3 RGD of Braced Excavation

For a braced excavation in clay using a diaphragm wall, the length of the wall (L), the thickness of the wall (t), the vertical spacing of the struts (S), and the strut stiffness (EA) are the design parameters. In the context of robust design, the goal is to

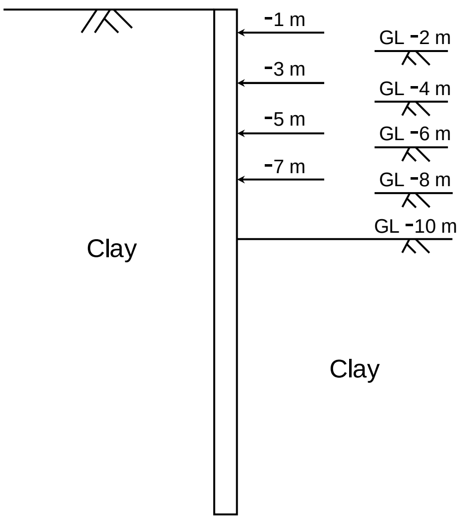

Fig. 11 Schematic illustration of braced excavation design example (modified after Juang *et al.* 2013b)

derive a satisfactory design by selecting a proper set of design parameters (L, t, S, EA) so that the system response, in the form of the maximum wall deflection, is insensitive to, or robust against the variation in noise factors (s_u / σ'_v, k_h / σ'_v, q_s). Of course, all the safety requirements have to be satisfied, and the construction cost has to be justified.

In this particular example of braced excavation in a uniform clay layer, the length of the wall L typically ranges from 20 m to 30 m with an increment of 0.5 m, and the thickness of wall t ranges from 0.5 m to 1.3 m with an increment of 0.1 m. The strut stiffness EA typically assumes a stiffness value from one of the five strut layout options: H300, H350, H400, 2@H350 and 2@H400 (note: 2@H400 means two H400 struts at the same level). The vertical spacing of the struts S typically assumes a value from one of the four choices: 1.5 m, 2 m, 3 m and 6 m. Based on the combination of the four design parameters (L, t, S, EA), there are totally 3780 possible discrete designs in the design space.

For each design in the design domain, PEM is used to evaluate the mean and standard deviation of the maximum wall deflection caused by variation in noise factors, and the cost of the supporting system is estimated using the procedure documented in Juang *et al*. (2013b). In this study, a multi-objective optimization is performed considering robustness (based on the variation in the predicted maximum wall deflection) and cost as the objectives and the safety requirements (including both stability and serviceability) as the constraints. The configuration for this multi-objective optimization is shown in Fig. 12. After the optimization, 25 "unique" designs are selected into the final Pareto Front, as shown in Fig. 13.

The Pareto Front in Fig. 13 describes a trade-off relationship between robustness and cost for decision making in braced excavation design. The designer can select the most preferred design based on the specified target cost/robustness level. For example, if the threshold budget for a supporting system is 1×10^6 USD, the design with least standard deviation of the wall deflection within the cost level on Pareto Front will be the most preferred design. This design has the following parameters: t = 0.8 m, L = 20 m, S = 1.5 m and EA = stiffness of H400 strut.

Given: L_E = 40 m (length of excavation)

B_E = 25 m (width of excavation)

H_f = 10 m (final excavation depth)

Find Design Parameters:

t (wall thickness), L (wall length)

S (strut spacing), EA (strut stiffness)

Subject to Constraints:

$t \in \{0.5\text{ m}, 0.6\text{ m}, 0.7\text{ m}, \ldots, 1.3\text{ m}\}$

$L \in \{20\text{ m}, 20.5\text{ m}, 21\text{ m}, \ldots, 30\text{ m}\}$

$S \in \{1.5\text{ m}, 2\text{ m}, 3\text{ m}, 6\text{ m}\}$

$EA \in$ {H300, H350, H400, 2@H350, 2@H400}

Mean factor of safety for push-in and basal heave $\geq$ 1.5 (Ou 2006)

Mean maximum wall deflection $\leq$ 7 cm (0.7%H_f, PSCG 2000)

Objective:

Minimizing the std. dev. of the maximum wall deflection (cm)

Minimizing the cost for the supporting system (USD)

Fig. 12 Formulation of multi-objective optimization for design of braced excavation (modified after Juang *et al.* 2013b)

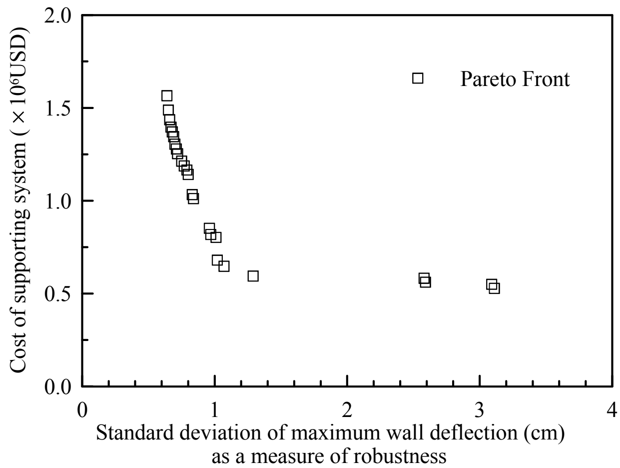

Fig. 13 Pareto Front for design of braced excavation

Although the Pareto Front provides a valuable tool the designer may use to make a more informed decision, the designer may prefer a single most optimal design instead of a set of optimal designs. Thus, a knee point concept based on gain-sacrifice relationship is further used to refine the decision making that seeks a single most preferred design. The knee point is defined as the point on the Pareto Front where any departure from this point requires a large sacrifice in one objective to achieve a small gain in the other objective. Based on the normal boundary intersection method (Deb *et al*. 2011), the knee point can be indentified as the point on the Pareto Front that has the maximum distance from the boundary line. The boundary line is the line that connects two boundary points on the Pareto Front.

With the normal boundary intersection method, the knee point in Fig. 13 is readily identified, as shown in Fig. 14. This knee point (optimal design) has the following parameters: t = 0.6 m, L = 20 m, S = 1.5 m and EA = stiffness of H400 strut with a cost of 0.68×10^6 USD. As shown in Fig. 14, below this cost level, a slight gain in cost reduction requires a large sacrifice in design robustness (as reflected by a markedly increase in the variation of the maximum wall deflection). Above this cost level, a slight gain in improved robustness requires a large increase in cost, rendering it cost inefficient (Juang *et al.* 2013b).

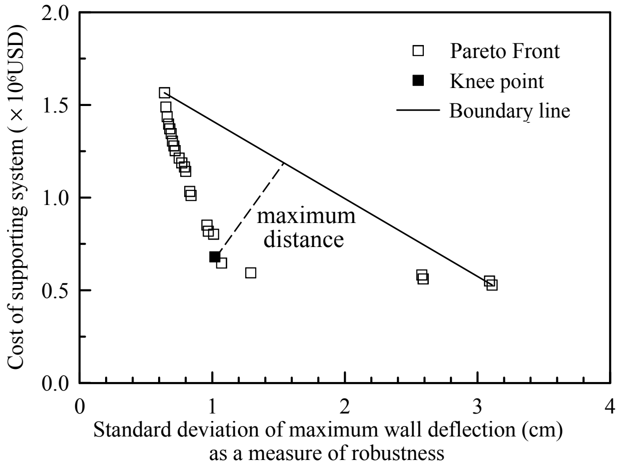

Fig. 14 Knee point identification for design of braced excavation

7. CONCLUSIONS

In this paper the authors present their novel Robust Geotechnical Design (RGD) methodology and its applications in several geotechnical problems including design of drill shaft, shallow foundation, and braced excavation. The purpose of this proposed RGD approach is to reduce the effect of uncertainties in the noise factors (*e.g.*, parameter and model uncertainties) by carefully adjusting the design parameters. Within the RGD framework, a multi-objective optimization is performed to identify optimal designs that are both cost-efficient and robust, while satisfying the safety requirements. Through this optimization, a Pareto Front is derived, which usually describes a trade-off relationship between cost and robustness at a given safety level. The derived Pareto Front provides a valuable tool for the designer to make a more informed design decision.

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LIST OF SYMBOLS

β_T	=	target reliability index
β	=	computed reliability index
β_β	=	feasibility robustness index
c'	=	effective cohesion
$\mathbf{d}$	=	vector of design parameters
F_{50}	=	50-year return period load
G	=	permanent load component
φ'	=	drained friction angle
K_0	=	coefficient of earth pressure at rest
k_h / σ'_v	=	normalized modulus of horizontal subgrade reaction
μ_p	=	mean of computed failure probability
p_T	=	target failure probability
p_f	=	computed failure probability
P_0	=	confidence probability
Q_{ULS}	=	ULS compression capacity
Q_{SLS}	=	SLS compression capacity
Q	=	transient load component
q_s	=	surcharge behind the wall
s_u / σ'_v	=	normalized undrained shear strength

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