

# VARIANCE-BASED SENSITIVITY ANALYSIS ON PSHA

Min-Hao Wu<sup>1</sup>, Jui-Pin Wang<sup>2\*</sup>, and Chia-Ying Sung<sup>3</sup>

## ABSTRACT

Probabilistic Seismic Hazard Analysis (PSHA) was commonly used in the past decade for developing site-specific earthquake-resistant designs. PSHA is a function-of-random-variable calculation in which earthquake magnitude, source-to-site distance, and the error term of Ground Motion Prediction Equation (GMPE) are the three random variables considered in the analysis. This paper introduces the first series of Variance-Based Sensitivity Analysis on PSHA, quantifying the sensitivity for the three variables. It shows that GMPE (or GMPE's error term) plays the most crucial role in PSHA, or the result of PSHA is most susceptible to GMPE. Given that its sensitivity was as high as 76% (24% for the other two parameters combined), new strategies for conducting a PSHA project more effectively were also proposed in this paper.

**Key words:** PSHA, variance-based sensitivity analysis, GMPE.

## 1. INTRODUCTION

Probabilistic Seismic Hazard Analysis (PSHA) is a method for estimating the annual rates of ground motion exceedances at a site using a Ground Motion Prediction Equation (GMPE) as the performance function for such a function-of-random-variable analysis. Its original framework was proposed in the late 1960s (Cornell 1968). As its title indicates, PSHA is a *probabilistic* analysis or a function-of-random-variable analysis, and its concepts are the same as other engineering probabilistic assessments. More technical details about probabilistic analyses and PSHA were given in the Methodology of this paper.

Many PSHA-related studies and papers have been conducted and reported in the past decades. Generally speaking, most of them belong to one of the three categories:

1. Case study (Cheng *et al.* 2007; Mucciarelli *et al.* 2008; Benito *et al.* 2010; Lin *et al.* 2011; Stirling *et al.* 2011; Barani *et al.* 2020; Das *et al.* 2020; Peñarubia *et al.* 2020; Kowsari *et al.* 2021);
2. Comments and debates on the method's robustness (Castaños and Lomnitz 2002; Bommer and Abrahamson 2006; Wang 2011; Musson 2012a, 2012b; Panza *et al.* 2014; Wong 2014; Mulargia *et al.* 2017);
3. New and modified approaches (Papaspiliou *et al.* 2012; Alimoradi and Beck 2015; Bayliss 2016; Payne *et al.* 2017; Meirova *et al.* 2018; Nas *et al.* 2020; Ordaz *et al.* 2022; Wang *et al.* 2022; Motaghed *et al.* 2023).

Besides, in terms of its “daily” applications, PSHA has been part of the “standard procedure” for performance-based, earthquake-resistant designs, especially for the safety-related facilities of nuclear power plants. To our knowledge, at least two technical

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<sup>1</sup> Professor, Department of Civil and Environmental Engineering, National University of Kaohsiung, Kaohsiung, Taiwan.

<sup>2\*</sup> Professor (corresponding author), Department of Civil Engineering, National Central University; 300 Zhongda Rd., Zhongli, Taoyuan, 320317, Taiwan (e-mail: jpwang@ncu.edu.tw).

<sup>3</sup> Ph.D. Candidate, Department of Civil Engineering, National Central University, Taoyuan, Taiwan.

references have been implemented using PSHA for earthquake-resistant designs (USNRC 2007; MOI 2022).

For a function-of-random-variable analysis (or probabilistic analysis), each expansionary variable ( $X_i$ ) makes different contributions to the variance of the dependent variable ( $Y$ ). The one that contributes more is considered of more “importance” to the function-of-random-variable calculation because it dominates the probability distribution of  $Y$ . Say, the performance function of the function-of-random-variable analysis as  $Y = X_1 + 50X_2$ , and the variance of  $Y$  will be mainly contributed from  $X_2$ , or the probability distribution of  $Y$  is dominated by  $X_2$ .

To quantify such effects, a measure called “sensitivity” from the Variance-Based Sensitivity Analysis (VBSA) was proposed (Saltelli *et al.* 2008). VBSA aims to characterize the respective sensitivity of the explanatory variables in a function-of-random-variable analysis quantitatively. Using the previous  $Y = X_1 + 50X_2$  as an example, the individual sensitivities of  $X_1$  and  $X_2$  are 0.04% and 99.96% based on VBSA, considering that both follow the uniform distribution from 0 to 1. Intuitively, we realized  $X_2$  contributed more to the variance of  $Y$  in this example, but VBSA provided the quantitative numbers. More technical details about VBSA were also given in the Methodology of this paper.

For the first time, this novel study used VBSA to investigate the sensitivity of the three random variables considered in PSHA. Based on the findings, engineers should spend more effort on those with higher sensitivity when conducting a PSHA project in the future, the most significant novelty, impact, and contribution from this study.

## 2. METHODOLOGY

### 2.1 Variance-Based Sensitivity Analysis

For a function of random variables as  $Y = f(X_S)$ , the variance of  $Y$  is contributed from *i*) the variance of  $X_i$  and *ii*) the “role” of  $X_i$  in the function. For example, given  $Y = X_1 + X_2$  and the respective variances of  $X_1$  and  $X_2 = 100$  and 1, the variance of  $Y$  will be mainly contributed from  $X_1$ . By contrast, given  $Y = 100X_1 + X_2$  and both having a variance of 1, the variance of  $Y$  will also be principally contributed from  $X_1$ .

The Variance-Based Sensitivity Analysis was proposed to quantify each explanatory (input) variable's contribution to the variance of  $Y$  in a function of random variables (Saltelli *et al.* 2008). Specifically, the sensitivity of each explanatory variable,  $S_{Xi}$ , was defined as follows:

$$S_{Xi} = \frac{V_{Y\_Xi}}{V_{Y\_total}} \quad (1)$$

where  $V_{Y\_total}$  denotes the variance of  $Y$  on the condition that all explanatory parameters are considered a random variable in the calculation;  $V_{Y\_Xi}$  denotes the variance of  $Y$  with only  $X_i$  regarded as a random variable while the others are viewed as a constant represented by their mean values.

Here is an example to explain the sensitivity calculation and interpretation. Given that  $Y = 100X_1 + X_2 + X_3$  and the mean and SD (standard deviation = variance<sup>0.5</sup>) of the three explanatory variables were (0, 1), (0, 100), and (0, 1), thus  $V_{Y\_total}$  is 10101 with all of them as random variables, and  $V_{Y\_X1}$ ,  $V_{Y\_X2}$ , and  $V_{Y\_X3}$  are 10000, 100, and 1, respectively. As a result, their respective sensitivities are 99%, 0.99%, and 0.01%, or the variance of  $Y$  is dominated by  $X_1$  and is almost irrelevant to  $X_3$  (its sensitivity = 0.01%). For example, as the SD of  $X_3$  increases 100% from 2 to 1,  $V_{Y\_total}$  only increases 0.01% from 100.503 to 100.519. By contrast, when the SD of  $X_1$  also increases 100% from 2 to 1,  $V_{Y\_total}$  can increase 99% from 100.503 to 200.252.

## 2.2 PSHA, Total-Probability Calculation, and MCS

As mentioned previously, PSHA is also a function-of-random-variable analysis, considering earthquake magnitude, source-to-site distance, and the error term of GMPE as random variables to calculate a given ground motion exceedance probability, like  $\Pr(\text{PGA} > 0.1 \text{ g}) = 20\%$ . Note that this exceedance probability is induced by *one* earthquake of a particular seismic source. Given that this seismic source would produce  $v$  earthquakes per year, the annual rate of the motion of exceedance,  $\lambda_{\text{PGA} > 0.1\text{g}}$ , will be equal to  $0.1v$  per year.

Because it is difficult to solve a function of random variables using the analytical algorithm, alternative ones were proposed, such as Monte Carlo Simulation (MCS) and total probability calculation. To the best of our knowledge, MCS is most commonly

used in solving a function of random variables, and sometimes, it is used as the benchmark to verify new algorithms when the analytical solution can hardly be developed.

In a nutshell, the MCS method takes advantage of the “power” of randomization, and the occurrence probability of a particular event is equal to the ratio of the occurrence frequency of the event to the total randomization trials, also known as the sample size of MCS. The equation is as follows:

$$\Pr(X) = \frac{N_X}{N_{\text{total}}} \quad (2)$$

where  $N_X$  denotes the occurrence frequency of  $X$  among the total randomization trials, denoted as  $N_{\text{total}}$ .

Nevertheless, PSHA does not use the analytical algorithm nor MCS for calculating the ground motion exceedance probability, *i.e.*,  $\Pr(\text{PGA} > y^*)$ . Instead, it uses the total probability calculation as follows:

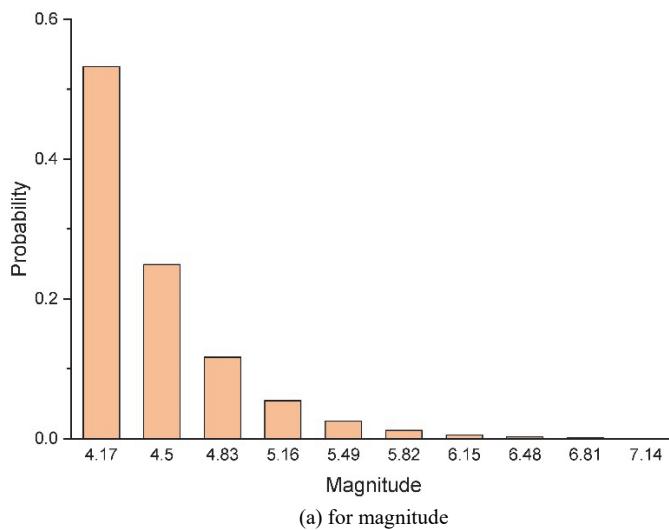
$$\Pr(\text{PGA} > y^*)$$

$$= \sum_{i=1}^{N_M} \sum_{j=1}^{N_D} \Pr(\text{PGA} > y^* | m_i, d_j) \times \Pr(M = m_i) \times \Pr(D = d_j) \quad (3)$$

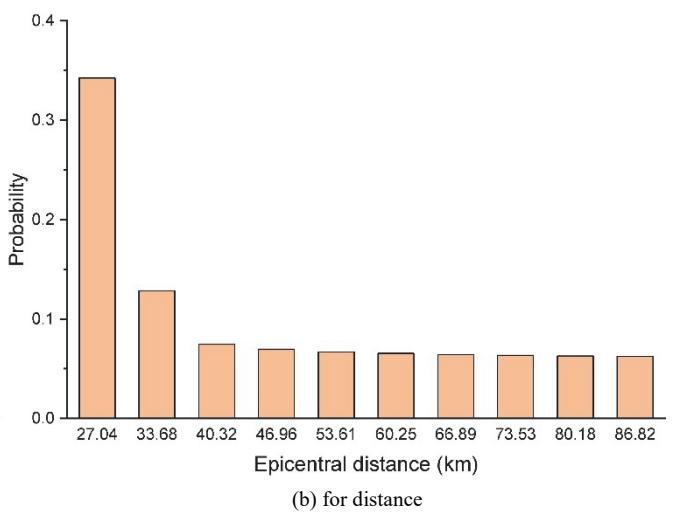
where  $\Pr(M = m_i)$  is the probability density as earthquake magnitude equal to  $m_i$  based on the given probability density function, and similar for  $\Pr(D = d_j)$ .  $\Pr(\text{PGA} > y^* | m_i, d_j)$  is the ground motion exceedance probability for  $M = m_i$  and  $D = d_j$  ( $y^*$  denotes a real value).  $N_M$  and  $N_D$  are the data bins in the magnitude and distance probability distributions (For example, they are 10 for those shown in Fig. 1). Since this algorithm is a derivative of the total probability theorem, it is called the total probability algorithm for solving a function of random variables.

Most importantly, regardless of which algorithm is used for functions of random variables, the result must be the same. In other words, when a new algorithm cannot produce the same results as those from the analytical calculation or MCS, then the algorithm is not robust.

However, although the total probability calculation and MCS can produce the same ground motion exceedance probability (*i.e.*,  $\Pr(\text{PGA} > y^*)$ ) with PSHA, the standard deviation of PGA cannot be easily calculated with the total probability algorithm. As a result, we use MCS in this study because we need to compute the variance



(a) for magnitude



(b) for distance

Fig. 1 Probability distributions of the benchmark example

( $= \text{SD}^2$ ) of PGA for quantifying the respective sensitivity of earthquake magnitude, source-to-site distance, and the error term of GMPE.

### 2.3 PSHA Benchmark Example

A PSHA example in the textbook (Kramer 1996) was used in this study. In addition to the demonstration of using the Variance-Based Sensitivity Analysis on PSHA, utilizing the textbook example can increase the transparency and accuracy of the results by comparing ours to those given in the textbook.

Figure 1 shows the magnitude and distance probability distributions for this PSHA example. Besides, other input data include:

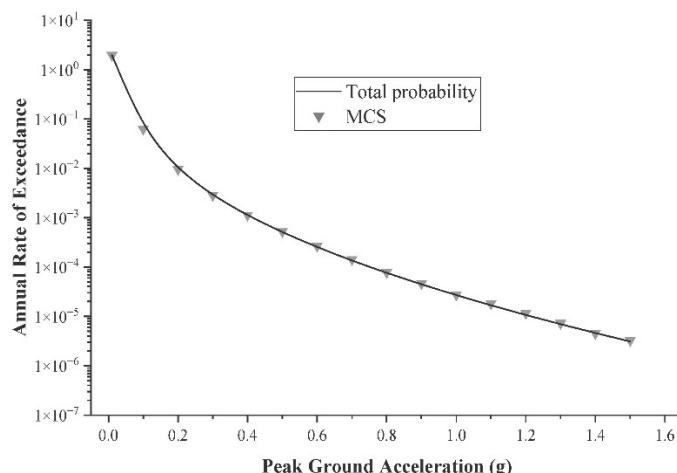
1.  $a$ -value of the Gutenberg-Richter recurrence law = 4.4;
2.  $b$ -value of the Gutenberg-Richter recurrence law = 1;
3. maximum magnitude ( $M_{\max}$ ) = 7.3;
4. minimum magnitude ( $M_0$ ) = 4;
5. GMPE as  $\ln(\text{PGA}) = 6.74 + 0.859 M - 1.8 \ln(D + 25) \pm 0.57$ ; where PGA in g, distance ( $D$ ) in km, and the standard deviation of the error term is 0.57.

With the input data, Fig. 2 shows the result given in the textbook. Explicitly, it was via the total probability calculation (Eq. (3)).

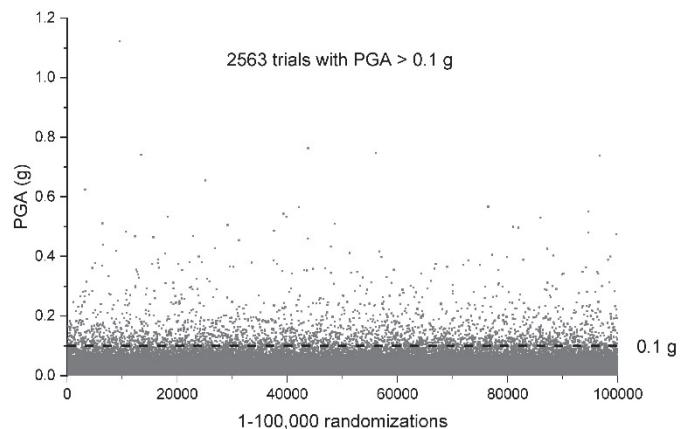
Figure 3 shows the “raw data” from MCS for solving this exact function of random variables. Accordingly, 2563 randomizations produced a PGA greater than 0.1 g among the 100,000 trials (sample size), so the exceedance probability was 2.563% via the MCS. Because the earthquake annual rate is 2.5 given  $a$ -value = 4.4,  $b$ -value = 1,  $M_{\max}$  = 7.3, and  $M_0$  = 4, the annual rate of  $\text{PGA} > 0.1 \text{ g}$  equals 0.064 per year ( $= 2.563\% \times 2.5$ ). Similarly, we can compute other hazard levels and develop the hazard curve (Fig. 2) based on the same pool of data from the MCS (Fig. 3).

The hazard curve from the MCS is also shown in Fig. 2. As expected, it is nearly identical to that given in the textbook via total probability calculation. This certified that a PSHA-type function of random variables can be solved with MCS or total probability calculation as for any other function-of-random-variable problems.

Nevertheless, based on the 100,000 data from MCS, we can efficiently compute the variance of PGA in this function of random variables, which cannot be achieved with total probability



**Fig. 2** The result of the PSHA benchmark example



**Fig. 3** The “raw” data from the MCS

calculation. With the variance of PGA calculated, we can compute the respective sensitivity of earthquake magnitude, distance, and the error term of GMPE using the Variance-Based Sensitivity Analysis.

## 3. RESULTS

The calculations show that  $V_{\text{PGA\_total}}$  is 0.000904 g for the PSHA example, which is the variance of PGA with all of the three parameters considered as random variables. By contrast, it shows that  $V_{\text{PGA\_M}}$  is 0.000118 g, which is the variance of PGA when magnitude was regarded as a random variable only, while the other two were constants represented by their mean values. Similarly, it shows that  $V_{\text{PGA\_D}}$  and  $V_{\text{PGA\_e}}$  are 0.000079 g and 0.000707 g, respectively. Note that  $D$  and  $e$  are distance and the GMPE’s error term.

Based on the values of  $V_{\text{PGA\_total}}$ ,  $V_{\text{PGA\_M}}$ ,  $V_{\text{PGA\_D}}$ , and  $V_{\text{PGA\_e}}$ , we used Eq. (1) to calculate the sensitivity of the three parameters in the PSHA. It shows that the sensitivity of magnitude, distance, and the GMPE’s error term is 13%, 8.7%, and 78.3%, respectively. With the sensitivity of the GMPE’s error term over 75%, the inference is that the result of the PSHA is most susceptible to this parameter.

### 3.1 Different Magnitude-Distance Scenarios

The result and finding above are associated with the specific setup: the given magnitude and distance probability distributions shown in Fig. 1. To certify the finding, we conducted additional calculations using different magnitude and distance probability distributions. Specifically, two more magnitude probability distributions were used, governed by  $b$ -values of 0.8 and 1.2. Note that the original magnitude distribution is controlled by  $b$ -value = 1.0.

Similarly, two more distance probability distributions were used, which shared the same distribution pattern as the original but with the mean value increasing from 56 km to 106 km and to 156 km. Relatively speaking, the additional two can be considered moderate and far seismic sources compared to the original as a near seismic source.

Figure 4 shows the sensitivity of the GMPE’s error term (or GMPE) on the nine conditions ( $= 3 \times 3$ ). Regardless of which magnitude and distance distributions are concerned, the sensitivity of this parameter was above 70% (with one exception). In other

words, GMPE indeed plays the most crucial role in the PSHA, in relative to earthquake magnitude and distance.

By contrast, Figs. 5 and 6 show the sensitivity of magnitude and distance on the exact nine conditions. On average, the sensitivities of magnitude and distance are about 20.3% and 5.5%, respectively. Furthermore, it was also found that the distance's sensitivity was lowest for the far seismic source with an average source-to-site distance of about 156 km. For example, the

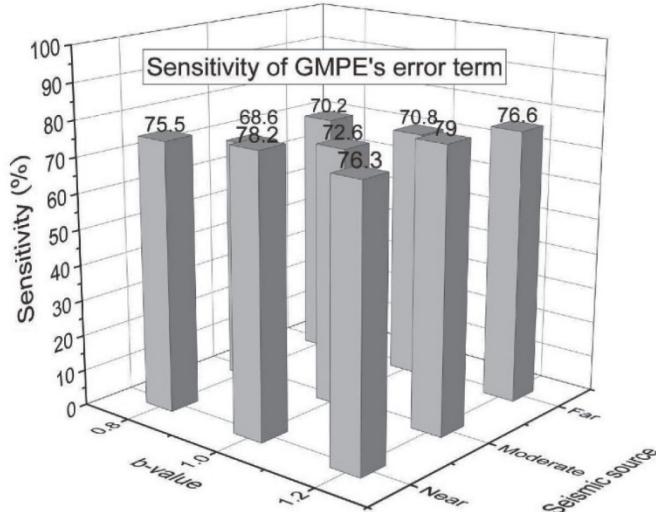


Fig. 4 The sensitivity of GMPE's error term on nine different conditions

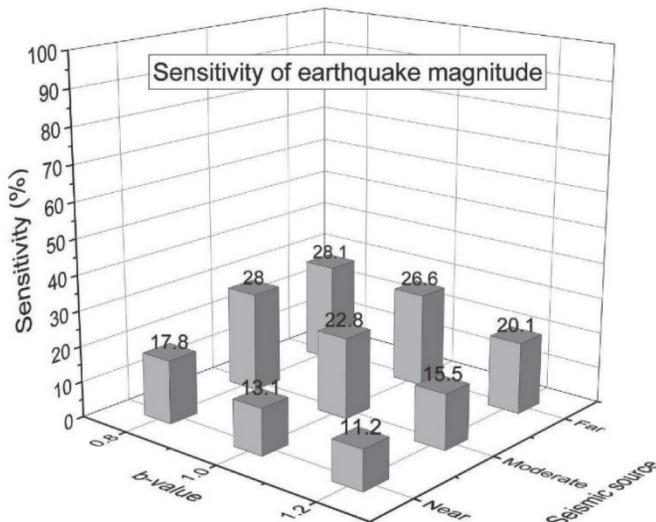


Fig. 5 The sensitivity of magnitude on nine different conditions

Table 1 The four GMPEs used in this study

Ground motion prediction equation	Reference	Local/Non-local*
$\ln \text{PGA} (\text{g}) = -2.822 + 1.076M - 1.777 \times \ln(D + 0.3828e^{0.583M}) \pm 0.549$	Gao et al. (2021)	Local
$\ln \text{PGA} (\text{g}) = 6.74 + 0.859M - 1.8 \times \ln(D + 25) \pm 0.57$	Cornell et al. (1979)	Non-local
$\ln \text{PGA} (\text{g}) = 0.125 + 12.86M - 1.133 \times \ln D \pm 0.69$	Mezcua et al. (2008)	Non-local
$\ln \text{PGA} (\text{g}) = 0.4648 + 0.9125M - 0.5 \times \ln D - 0.0118D \pm 0.77$	Garcia-Soto and Jaimes (2017)	Non-local

\* Local: model developed with data only from Taiwan

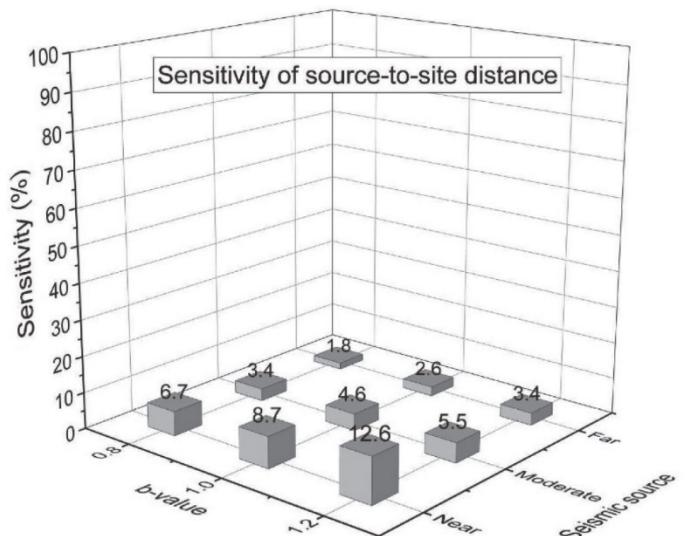


Fig. 6 The sensitivity of source-to-site distance on nine different conditions

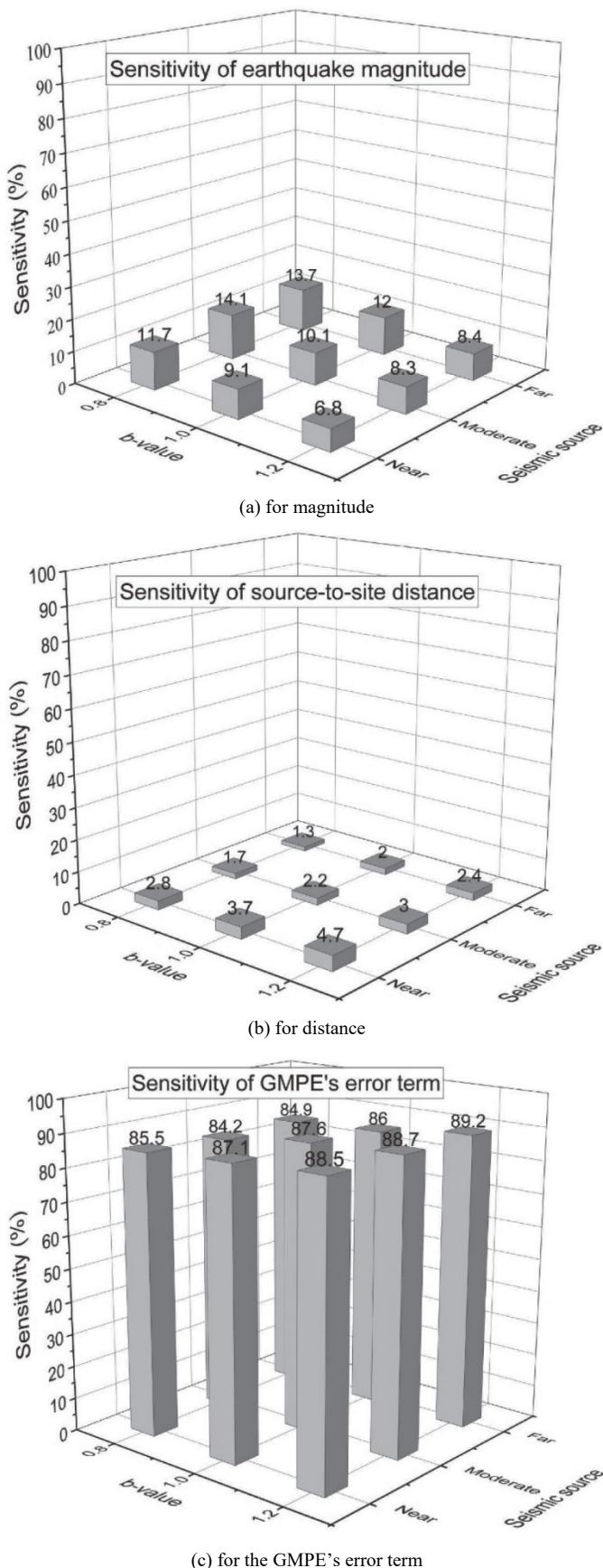
distance's sensitivity can be as low as 1.8% for such a far seismic source with the magnitude distribution controlled by  $b$ -value = 0.8.

### 3.2 Different GMPEs

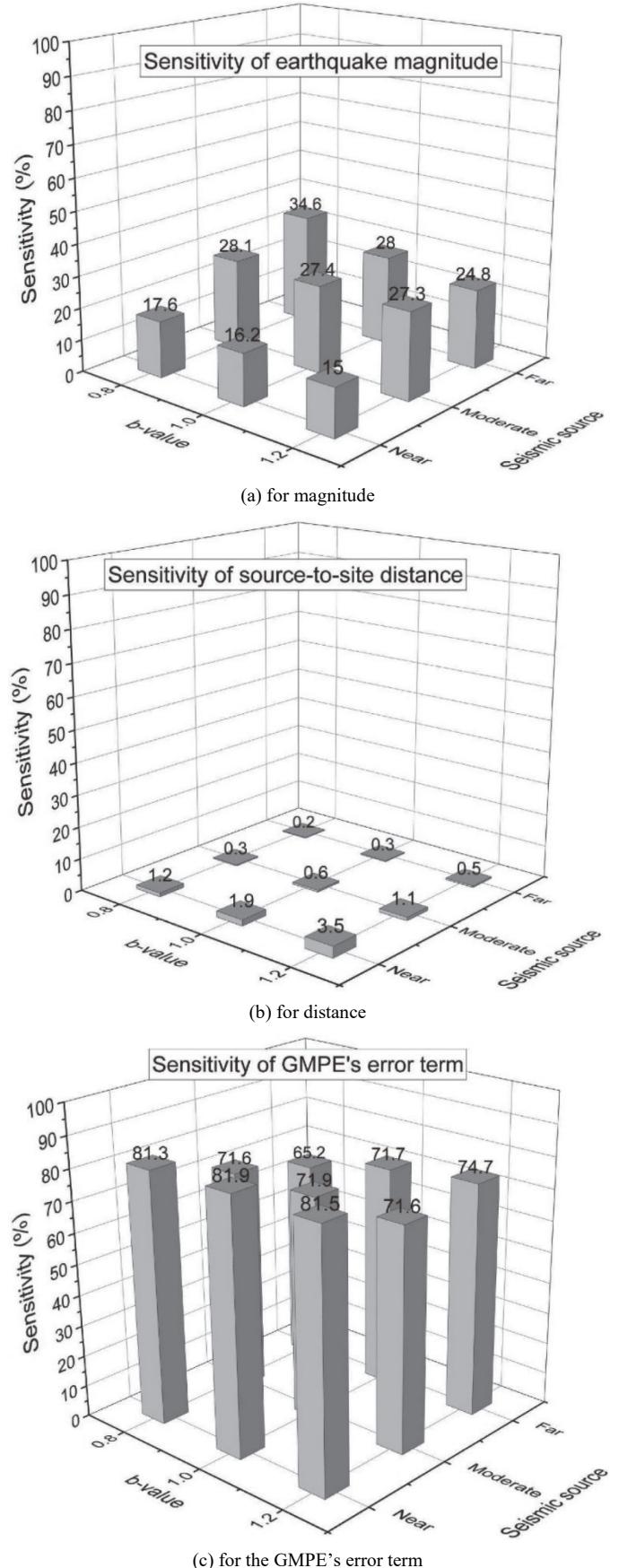
We used three additional GMPEs and repeated such sensitivity analyses to further certify the finding. In addition to the original GMPE used by the textbook example, the three additional GMPEs (Mezcua et al. 2008; Garcia-Soto and Jaimes 2017; Gao et al. 2021) are summarized in Table 1.

As the previous plots, Figs. 7 to 9 shows the sensitivity of magnitude, distance, and (GMPE's) error term on nine magnitude-distance combinations using the three GMPEs. The same trend appears in the 27 additional sensitivity analyses: i) GMPE's error term or GMPE plays the most critical and dominant role in PSHA; ii) the distance's sensitivity decreases as the source-to-site distance increases. For example, when the GMPE developed by García-Soto and Jaimes (2017) was used, the sensitivity of the GMPE's error term can be as high as 86.9% on average, and the distance's sensitivity gradually decreased from near seismic sources to far seismic sources.

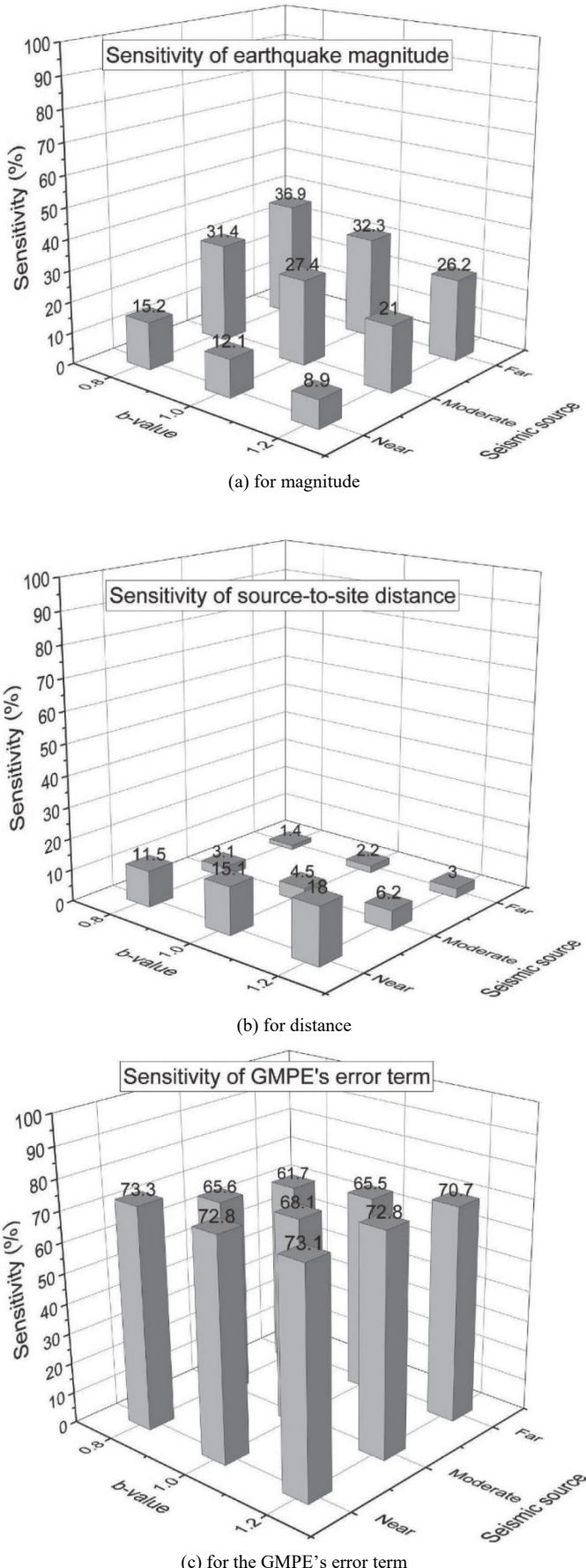
From the 36 sensitivity analyses on the nine magnitude-distance combinations using the four GMPEs, we calculated the average sensitivity of magnitude, distance, and GMPE's error term: it is 76.2% for GMPE's error term, 19.7% for magnitude, and 4.1% for distance. In summary, GMPE's error term plays the most critical and dominant role in PSHA, compared to the other two variables (*i.e.*, magnitude and distance) in PSHA.



**Fig. 7** Results of sensitivity analysis using the García-Soto-2017 GMPE (see Table 1)



**Fig. 8** Results of sensitivity analysis using the Mezcua-2008 GMPE (see Table 1)



**Fig. 9** Results of sensitivity analysis using the Gao-2021 GMPE (see Table 1)

## 4. DISCUSSION

### 4.1 Verification

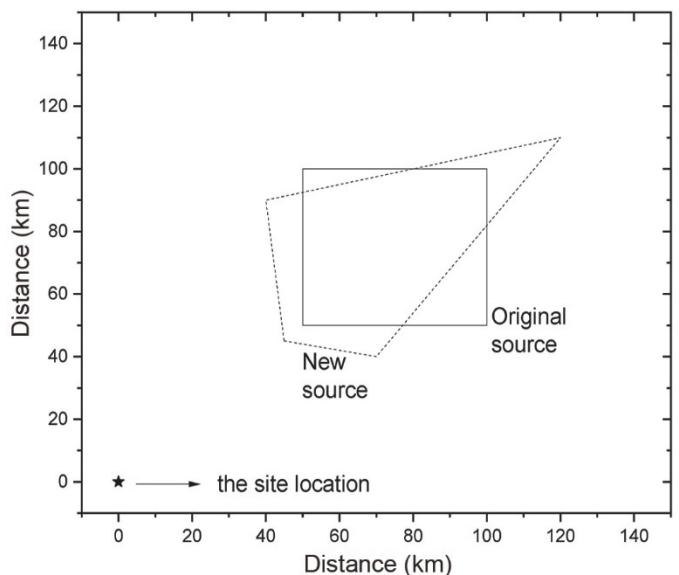
Based on the findings from this research, GMPE (or GMPE's error term) plays the most critical and dominant role in PSHA, while distance has very little influence on the results, especially for a far seismic source. This implies that the boundary of a seismic source does not matter too much in such a function-of-random-variable calculation. To verify this, additional analyses were conducted and shown in the following.

Figure 10 shows the boundary of the original seismic source, along with the new one whose boundary is different than the original. Accordingly, Fig. 11 shows the respective distance probability distributions used in the PSHA calculations to examine such an impact on the outcome.

Figure 12 shows the hazard curves of the two PSHAs associated with the two (far) seismic sources, with the remaining input data identical. It shows that the results are similar as expected because distance has a very low sensitivity in PSHA based on this research. For example, the annual rates of  $\text{PGA} > 0.2 \text{ g}$  induced by the original and new seismic sources are  $8.13 \times 10^{-5}$  and  $1.31 \times 10^{-4}$ , respectively. The normalized difference (*i.e.*,  $(A - B) / A$ ;  $A > B$ ) between the two is 30% approximately.

By contrast, Fig. 13 shows the respective hazard curves of two PSHAs using two different GMPEs with other inputs identical. It shows that the difference is more significant compared to Fig. 12. Also using  $\text{PGA} > 0.2 \text{ g}$  as an example, the annual rates (of  $\text{PGA} > 0.2 \text{ g}$ ) estimated by using the Cornell-1979 and Gao-2021 GMPEs (see Table 1) are  $8.13 \times 10^{-5}$  and  $1.05 \times 10^{-6}$ , respectively. The normalized difference between the two is about 100%, which is about threefold compared to the previous case (Fig. 12).

Nevertheless, it was not surprising to have such an outcome, given that this research revealed that GMPE plays the most critical and dominant role in PSHA. On average, its sensitivity is as high as 76% based on the 36 Variance-Based Sensitivity Analyses.



**Fig. 10** The boundary of two seismic sources

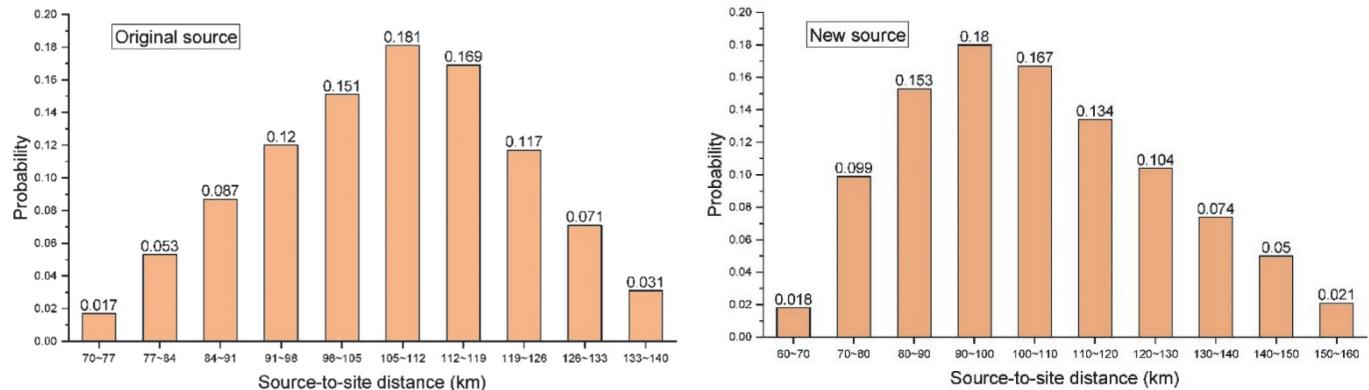


Fig. 11 The source-to-site distance probability distributions: (a) for the original source; (b) for the new one (see Fig. 10)

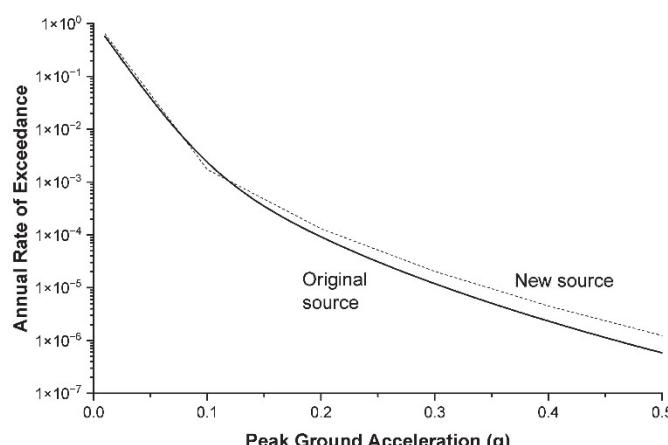


Fig. 12 The hazard curves associated with the two seismic sources (see Fig. 10)

#### 4.2 An Effective Approach Conducting a PSHA

Correct input data generate desirable results, so one prerequisite in an analysis is ensuring all the input data used are accurate. However, in earthquake science and research, it is difficult to achieve that; otherwise, earthquakes could have been predicted, and subsequent hazards could have been substantially mitigated (Geller *et al.* 1997). As to PSHA, several input data are “guesstimated” and subjective, such as the boundary or shape of a seismic source. Based on our experience, if the same pool of geological data is provided to ten research teams, the developed seismic source models will not be identical in our belief.

Nonetheless, this research conducted sensitivity analyses and revealed that the PSHA result is most susceptible to GMPE, with its sensitivity as high as 76% (*i.e.*, 24% for the other two parameters combined). As a result, a more effective manner when conducting a PSHA project is to spend proportional amounts of effort in characterizing respective input data in terms of their respective sensitivity. Therefore, we would spend less effort determining the shape of seismic sources than developing and selecting a GMPE that is more suitable for the study site. This does not mean a PSHA practitioner should “doodle” a seismic source model based on their “feelings” rather than scientific evidence. But, we suggest that the effort spent on “fine-tuning” a seismic source can be saved, while the extra effort can be spent on developing and/or selecting the most suitable GMPE for the site in the PSHA. This is a more

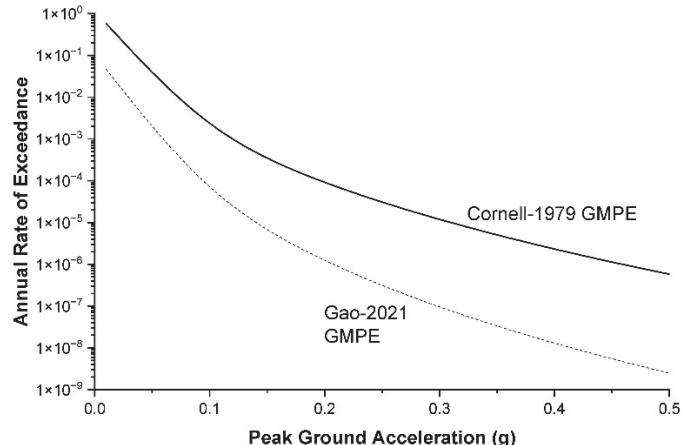


Fig. 13 The hazard curves using two different GMPEs

effective manner to conduct a PSHA, in our opinion, based on the findings from this study quantifying the sensitivity of magnitude, distance, and GMPE as 20%, 4%, and 76%, respectively.

#### 5. CONCLUSIONS

A series of Variance-Based Sensitivity Analyses were conducted on PSHA. It shows that GMPE (or GMPE’s error term) plays the most crucial and dominant role in PSHA. Its sensitivity is as high as 76%, and the sensitivity of the other two parameters combined is 24%. According to this research, a new strategy for conducting a PSHA project more effectively was also proposed in this paper.

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## DATA AVAILABILITY

All data and/or computer codes used/generated in this study are included in this paper.

## CONFLICT OF INTEREST STATEMENT

The authors have no conflict of interest to disclose.

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