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AN IMPROVEMENT IN THE DETERMINATION OF END BEARING CAPACITY OF DRILLED SHAFTS IN SAND

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ABSTRACT

There are many equations available in the literature using the results of CPT (cone penetration test) and SPT (standard penetration test) measurements to predict the end bearing capacity of drilled shafts in sand. However, there are few equations that use soil parameters, such as friction angle and elastic modulus, as input values. Also, these available equations usually overestimate the end bearing capacity, and at times show conflicting results with respect to the parameters they use. This paper describes a numerical procedure to overcome the above shortcomings. The results obtained in this study were compared with both experimental and numerical results available in the literature. A series of analyses is also conducted to assess the effects of various soil and pile parameters on the magnitude of end bearing capacity of piles embedded in sand. These parameters include diameter and length of pile, friction angle, and Poisson's ratio of soil. Based on the numerical analyses carried out in this study, a new equation is proposed to estimate the end bearing capacity of drilled shafts in sands. The results of the proposed equation are also compared with experimental data available in the literature on end bearing capacity of drilled shafts. The comparison shows better agreement of the suggested end bearing capacity equation with respect to other previously proposed methods.

Key words: Drilled shafts, bored piles, end bearing capacity, tip resistance, numerical modeling, sand.

1. INTRODUCTION

Piles are structural members generally used in situations where shallow foundations cannot withstand loads applied from the superstructure. There are two general methods for pile installation: driving them into the soil (driven pile) or boring a cylindrical hole in the ground and filling it with concrete along with a reinforcement cage (drilled shaft). In Europe, bored pile is a more common name than drilled shaft. For a given soil type and pile dimensions, bearing capacity of driven piles is usually greater than drilled shafts; however, drilled shafts have other advantages such as low cost and less vibration during installation. In this regard, determination of bearing capacity of drilled shafts is an important challenge for geotechnical engineers. Determination of the bearing capacity of drilled shafts includes two parts, tip resistance and shaft resistance. Tip resistance has a crucial role in bearing capacity of drilled shafts in sand so that some design guidelines are primarily based on tip resistance (Winterkorn and Fang 1975; Das 2010; Luo and Juang 2012; Stanton and Motamed 2017).

The literature consists of many equations for the determination of end bearing capacity of driven piles in sand. The input for these equations are either based on geotechnical parameters of soil measured in the laboratory (Meyerhof 1976; Janbu 1976) or based on values of in-situ measurements of SPT or CPT (Aoki and Velloso 1975; Meyerhof 1976; Decourt 1982). The literature also consists of equations to estimate tip resistance of drilled shafts in sandy soils based on in-situ measurements of SPT (Meyerhof 1976; Reese and Wright 1977; Decourt 1982; O'Neill and Reese 1999) or CPT (Aoki and Velloso 1975; Togliani 2008; Bustamante and Gianeselli 1982). However, there are few equations available in the literature that use laboratory measurements of geotechnical parameters of soil to estimate tip resistance of drilled shafts in sands. The most popular equation was initially proposed by Meyerhof (1963) and has developed over time by other researchers to take the form given in Eq. (1).

$$q_b = q'(N_q - 1)F_{qs}\, F_{qd}\, F_{qc} \tag{1}$$

In this equation, q' is the effective vertical stress at the pile tip, and N_q is bearing capacity factor. F_{qs}, F_{qd}, and F_{qc} are factors of shape, depth, and compressibility, respectively. Vesic (1973), Debeer (1970), Hansen (1970), and Chen and Kulhawy (1994) have suggested appropriate values for N_q, F_{qs}, F_{qd}, and F_{qc}, respectively (Kulhawy 1991; Kulhawy 2004). Among these parameters, the parameters F_{qd} and F_{qc} are discussed more in the present research, and are introduced below for convenience. (Eqs. (2) to (7))

$$F_{qd} = 1 + 2\tan\phi'\,(1-\sin\phi')^2\,\tan^{-1}\left(\frac{L}{D}\right) \tag{2}$$

$$F_{qc} = \exp\left\{(-3.8\tan\phi') + \left[\frac{(3.07\sin\phi')(\log_{10} 2I_{rr})}{1+\sin\phi'}\right]\right\} \le 1 \tag{3}$$

$$I_{rr} = \frac{I_r}{1 + I_r\Delta} \tag{4}$$

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$$I_r = \frac{E}{2(1+\nu)q' \tan \phi'} \tag{5}$$

$$\Delta = n\frac{q'}{P_a} \tag{6}$$

$$n = 0.005\left(1 - \frac{\phi' - 25°}{20°}\right) \tag{7}$$

In these equations, ϕ' is drained soil friction angle, E is soil elastic modulus, ν is soil Poisson's ratio, L and D are length and diameter of pile, I_r is rigidity index, I_{rr} is reduced rigidity index, P_a is atmospheric pressure, and q' is effective vertical stress at the base of the pile.

Brezantzev *et al.* (1961) have also proposed another equation to estimate end bearing capacity of drilled shafts in sand. Equation (8) below is based on Brezantzev *et al.*'s studies and its subsequent modifications. They provided Fig. 1 for the determination of parameter ω appearing in Eq. (8).

$$q_{b(net)} = q'(\omega N_q^* - 1) \tag{8}$$

$$N_q^* = 0.21e^{0.17\phi'} \tag{9}$$

Salgado (2008) has also suggested Eq. (10) to estimate tip resistance associated with a settlement of 10% of pile diameter.

$$q_{b,10\%} = 1.64P_a \times \exp[0.1041\phi_c + (0.0264 - 0.0002\phi_c)D_r] \times \left(\frac{\sigma'_h}{P_a}\right)^{(0.841-0.0047D_r)} \times [0.23\exp(-0.0066D_r)] \tag{10}$$

In this equation, ϕ_c is the critical-state friction angle, P_a is atmospheric pressure, and D_r is soil relative density in percent. Also, σ'_h is the horizontal effective stress at the pile tip.

The equations of Meyerhof (1963) and Brezantzev *et al.* (1961) are the well-known equations in which geotechnical parameters are used as input values to determine end bearing capacity of drilled shafts in sand. However, equations proposed by Meyerhof (1963) and Brezantzev *et al.* (1961) overestimate field measured values. The reason is that these equations are derived based on limit equilibrium analyses. This type of analyses assumes the pile experiences very large settlements. Therefore, these equations cannot estimate appropriate values of end bearing capacity in situations where there is a limit on the allowable settlement. Moreover, the effect of elastic modulus (E) and Poisson's ratio (ν) have not been considered in Brezantzev's and Salgado's equations. Also, the factors F_{qd} and I_r used in Meyerhof's relationship show that tip resistance is decreased if pile diameter (D) and Poisson's ratio of soil (ν) are increased. Furthermore, there is an inconsistency between Meyerhof's, Brezantzev's and Salgado's methods in determining the effect of pile diameter (D) on tip resistance of drilled shafts. More details on the effects of pile diameter on tip resistance are discussed in Section 5.2.

The present study aims to investigate the effects of pile dimensions (D and L) and soil parameters such as friction angle, modulus of elasticity, and Poisson's ratio (ν) on tip resistance and suggests a new equation for estimating end bearing capacity of drilled shafts in sand.

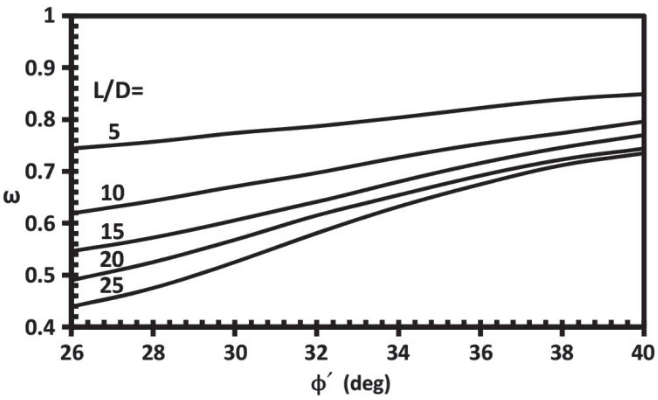

Fig. 1 Determination of ω from friction angle (Brezantzev *et al.* 1961)

2. DETERMINATION OF END BEARING CAPACITY OF DRILLED SHAFTS BASED ON CPT AND SPT VALUES

2.1 CPT Correlations

Aoki and Velloso (1975) recommended a method to estimate the end bearing capacity of piles from the results of dynamic penetration tests. Their method is based on the values of q_c measured in a static CPT test. By introducing certain empirically derived factors for different pile and soil types from load test results, some correlations were proposed to estimate the tip resistance of single piles. Equation 11 presents their correlation for end bearing capacity of drilled shafts embedded in sand.

$$q_b = \frac{q_{ca\,(tip)}}{3.5} \leq 15 \text{ MPa} \tag{11}$$

In this equation, $q_{ca\,(tip)}$ is arithmetic average of q_c around the tip of drilled shaft.

Also, Bustamante and Gianeselli (1982) proposed a method based on the analysis of 197 full-scale static load tests (including compression and uplift) from 48 sites on 96 pile foundations embedded in such varied soils as clay, silt, sand, gravel, weathered rock, mud, peat, weathered chalk, and marl. The deep foundations taken into account include both driven piles and drilled shafts. This method is commonly known as the Laboratoire Central des Ponts et Chaussées (LCPC) method or French method. The approach offers versatility in the variety and types of different deep foundation systems and geomaterials that can be accommodated. Equation (12) shows the suggested equation for drilled shafts embedded in sand.

$$q_b = 0.15\, q_{c\,(tip)} \tag{12}$$

In this equation, $q_{c\,(tip)}$ is the average of measured q_c values in a zone ranging from 1.5D below pile tip to 1.5D above pile tip (D is the pile diameter).

In addition, Togliani (2008) proposed a simplified direct CPT-based method for driven piles and drilled shafts, cylindrical as well as tapered piles. In particular, he combined years of his experience of designing and predicting the pile behavior with the LCPC method and certain previous suggestions for calculating the capacity of tapered piles to provide design equations for both shaft and tip resistances. Equation (13) shows his recommendation for both driven piles and drilled shafts embedded in sand (Niyazi and Mayne 2013).

$$q_b = \left(0.1 + 0.01\frac{L}{D}\right) q_{c\,(tip)} \tag{13}$$

In this equation, L and D are the length and diameter of drilled shaft, respectively. Also, $q_{c\,(tip)}$ is the average of q_c values in a zone ranging from $4D$ below the pile tip to $8D$ above the pile tip.

The above correlations are based on the value of q_c. Robertson and Campanella (1983) suggested an empirical relationship to be applicable to uncemented, unaged, moderately compressible quartz sands. This suggestion is based on the results of calibration chamber test. Equation (14) shows their proposed relationship. This Equation correlates the value of q_c to soil friction angle.

$$\tan\phi' = \frac{1}{2.68}\left[\log\left(\frac{q_c}{\sigma'_{v0}}\right) + 0.29\right] \tag{14}$$

In this equation, ϕ' represents drained friction angle of sand, and q_c is the measured tip resistance derived from mechanical cone penetrometers. Also, σ'_{v0} stands for the effective initial vertical stress at the pile tip.

2.2 SPT Correlations

Meyerhof (1976) proposed the following simple empirical relationship, Eq. (15), to estimate the tip resistance of drilled shafts.

$$q_b = 120\ N \tag{15}$$

where q_b is tip resistance with the unit of kPa, and N represents average standard penetration resistance (blows per foot) near the tip.

Reese and Wright (1977) and Decourt (1982) also proposed different relations to estimate the tip resistance of drilled shafts embedded in sand. Equations (16) and (17) present their suggested relations respectively.

$$q_b = 65\ N \tag{16}$$

$$q_b = 150\ N \tag{17}$$

O'Neill and Reese (1999), depending on whether the embedment depth of drilled shafts is shorter or longer than 10 m, recommend the following empirical equation:

$$q_b = \frac{L}{10}\,57.5\ N \le \frac{L}{10}\,2900\ \text{kPa} \qquad \text{for } L < 10\ \text{m} \tag{18}$$

$$q_b = 57.5\ N \le 2900\ \text{kPa} \qquad \text{for } L \ge 10\ \text{m} \tag{19}$$

where L is the embedment depth of drilled shaft (m).

O'Neill and Reese (1999) suggest that their equation is applicable when good practice is exercised for the installation and construction of drilled shafts, the bore hole is stabilized, boring is carried out according to the required dimensions, and concreting is performed correctly (Reese *et al.* 2006).

It can be observed that the estimation of tip resistance of drilled shafts presented in the above empirical relations in this approach is based only on the standard penetration number (N).

Schmertmann (1975) proposed a correlation between effective overburden pressure (σ'_{v0}), N, and ϕ'. Kulhawy and Mayne (1990) approximated this method in the form of Eq. (20). This Equation correlates the value of N to soil friction angle.

$$\tan(\phi') = \left[\frac{N}{12.2 + 20.3\left(\dfrac{\sigma'_{v0}}{P_a}\right)}\right]^{0.34} \tag{20}$$

3. NUMERICAL MODELING

In this study, FLAC^{2D}, which is a software based on finite difference method, has been used to numerically model the drilled shafts in homogenous sands. In this regard, the axisymmetric option has been selected for numerical modeling of drilled shafts. Based on several sensitivity analyses, height and width of the model were considered to be 2.5 times the pile length and 25 times the pile radius, respectively. These dimensions for model sizes (model height and width) are chosen so that the effects of boundaries on the results are minimized. The boundary condition for the side boundaries is chosen to be restrained against horizontal direction (X). The bottom boundary is restrained in both X and Y directions. In this study, the solid body is divided into a finite difference mesh composed of quadrilateral elements. Internally, FLAC subdivides each element into two overlaid sets of constant-strain triangular elements. Also, several analyses have been performed for element sizes of 2 cm × 2 cm, 5 cm × 5 cm, 10 cm × 10 cm, and 20 cm × 20 cm. The authors found that the results obtained based on element size of 20 cm × 20 cm are different from the other mentioned element sizes. The authors also found that the results of element sizes of 2 cm × 2 cm, 5 cm × 5 cm, and 10 cm × 10 cm were the same. Therefore, dimensions of the elements were selected to be 10 cm × 10 cm in the vicinity of the axis of symmetry, and become larger as their distance from the axis of symmetry are increased until the elements finally reach a size of 10 cm × 50 cm at the right boundary. The numerical analyses show that the results of this meshing are similar to the results for the case of all elements having a size of 10 cm × 10 cm.

The contact of the soil with pile tip and shaft is modeled by two interface elements. Both of the interfaces have the same parameters such as normal stiffness ($K_n = 3 \times 10^8$ N/m^3) and shear stiffness ($K_s = 1 \times 10^8$ N/m^3). These values were selected based on sensitivity analyses, and these values are in the range of values recommended in FLAC manual. Also, Yuan *et al.* (2004) found that the normal stiffness of soil-pile interface should be selected on the order of 10^8 N/m^3. In addition, the sensitivity analyses on the interface shear stiffness values showed that the results would not change if larger values of 1×10^8 N/m^3 are chosen. Also, the soil-pile interface friction angle ϕ_{int} is assumed to be equal to $0.8\phi'$, where ϕ' is the soil friction angle. Figure 2 shows the finite element model with its boundary conditions, location of interfaces and the elements used for the analyses in this study. In the numerical analyses procedure, the nodal points associated with drilled shaft are loaded by applying a vertical velocity. The present numerical

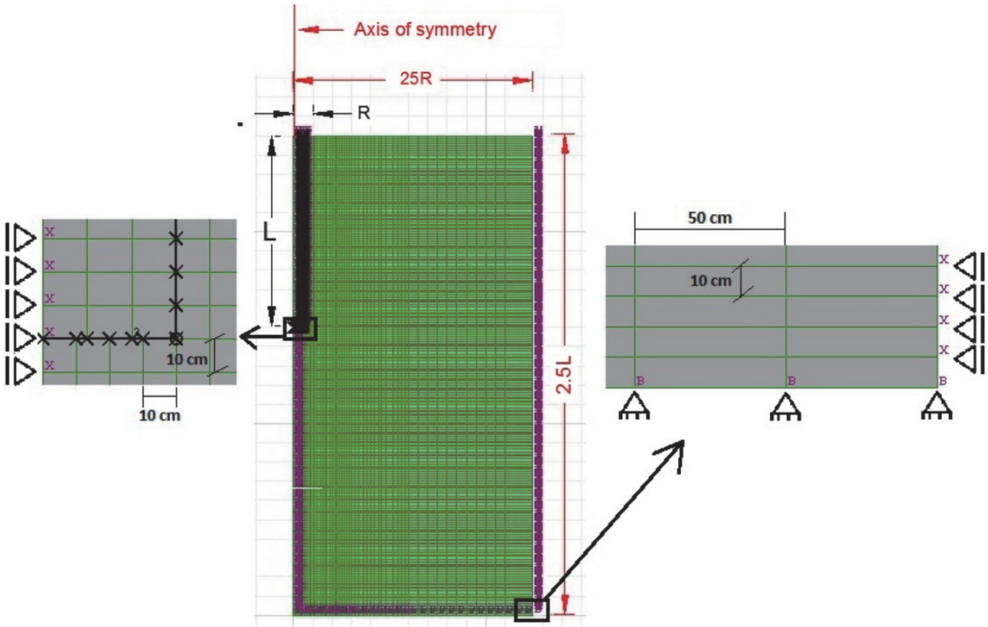

Fig. 2 A sample for the mesh and the boundary conditions used in the numerical modeling in this study

modeling has also discretized the pile into 4 noded elements. Also, nodal points associated with pile elements are fixed, meaning that the pile nodal points are not displaced in comparison to each other. Therefore, the pile is assumed to act as a rigid element.

In the present numerical modeling, the Mohr-Coulomb Elasto-Plastic constitutive law is used to model sand. Main parameters required for using Mohr-Coulomb criterion in sand are friction angle (ϕ'), dilation angle (ψ), elastic modulus (E), Poisson's ratio (ν) and soil unit weight (γ). For pile material, the linear elastic behavior is assigned with the parameters shown in Table 1.

Table 1 Parameters used to model the pile

Elastic modulus E (GPa)	Poisson's ratio ν	Unit weight γ (kN/m^3)
30	0.2	25

Dilation angle (ψ) is estimated using Eq. (21) below recommended in Plaxis manual (2016).

$$\psi = \phi' - 30° \geq 0 \tag{21}$$

Furthermore, Bowles (1996) recommended a value of 0.3 for Poisson's ratio (ν) for sand. The same assumption is made in this study.

4. VERIFICATION

In the present study, two field load tests have been used for verification of the numerical modeling results.

4.1 Case 1: Comparison of Numerical Results with Field Test

Mayne and Harris (1993) conducted a pile load test at Georgia Institute of Technology test site consisting mainly of sand with geotechnical parameters measured in the laboratory for different layers of the test site. A drilled shaft was loaded to reach a settlement of about 160 mm and its associated tip resistance was reported to be about 3000 kPa. Table 2 presents the values of friction angle (ϕ') and coefficient of lateral earth pressure at rest (K_0) for each layer considered by Lee and Salgado (1999). Dilation angle (ψ) for each layer is calculated based on Eq. (21). Also, elastic modulus (E) and unit weight (γ) are determined by engineering judgment and using Table 9 for the soil of the test site and Table 3 for drilled shaft. Figure 3 shows a comparison of the numerical model results with values measured in the field indicating good agreement between the numerical results obtained in this study and field values.

Table 2 Soil parameters of case 1 used in the numerical model

Depth (m)	K_0	ϕ' (deg)
0 ~ 1.8	0.44	34
1.8 ~ 3.9	0.44	34
3.9 ~ 5.9	0.4	37
5.9 ~ 7.9	0.46	33
7.9 ~ 9.9	0.47	32
9.9 ~ 11.9	0.47	32
11.9 ~ 13.9	0.41	36
13.9 ~ 14.9	0.38	38
14.9 ~ 45.0	0.41	36

Table 3 Drilled shaft properties used in case 1

L (m)	D (mm)	γ (kN/m^3)	E (GPa)	ν
16.8	760	25	30	0.2

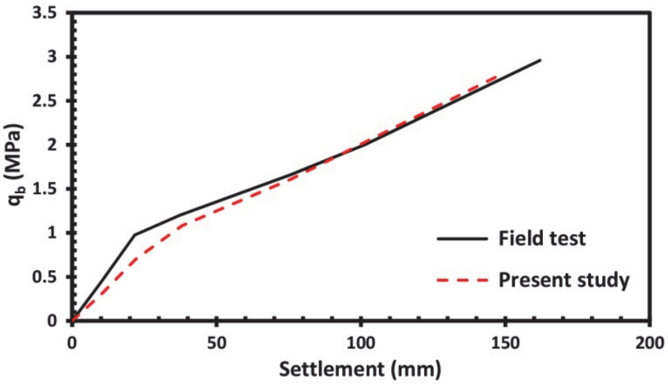

Fig. 3 Comparison of field test with the numerical model of Case 1

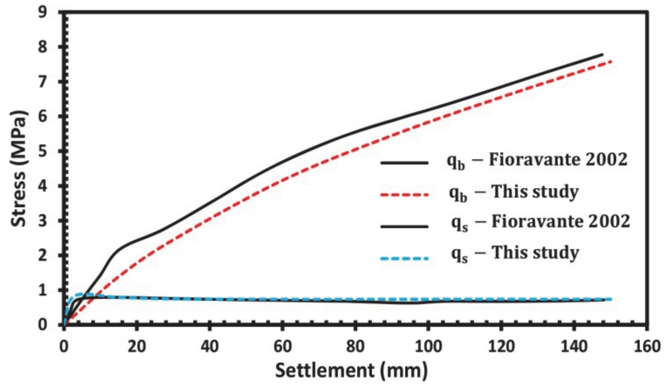

Fig. 4 Comparison of field and numerical model results

4.2 Case 2: Comparison of Numerical Results with Centrifuge Test

Fioravante (2002) performed a centrifuge test on Toyoura sand with a relative density of 93%. He recommended using values of 8 m and 300 mm as length and diameter of the pile, respectively. This study uses the recommendation of Fioravante (2002) for the length and diameter of the pile. Table 4 shows the properties of the soil which is used in this study. Figure 4 presents the comparison of the results of numerical modeling with the centrifuge test results. This figure also displays acceptable agreement between the results of the centrifuge test and the present study.

5. RESULTS AND DISCUSSIONS

5.1 Criteria for Tip Resistance Determination

In this study, more than 1500 numerical analyses are performed to numerically determine tip resistance of drill shafts in sand. The variable parameters are D, L, E, ν, γ, and ϕ'. Table 5 shows different values of pile and sand parameters used in this study. The results of field and laboratory tests (Figs. 3 and 4) show that the load-settlement curve does not reach any peak point of the tip resistance associated with the settlement of 20% pile diameter. It is therefore necessary to select a suitable criterion for the determination of ultimate values of tip resistance. Different researchers have recommended several criteria. Fleming *et al.* (2008) suggested that the displacement needed for full mobilization of end bearing capacity was in the range of 5 to 10% D, where D is the pile diameter. According to Whitaker and Cooke (1966), Franke (1989) and also Salgado (2008), failure load corresponds to a settlement of 10% D. Reese and Wright (1977) and O'Neil and Reese (1999) suggested the value of 5% for the ratio of settlement to diameter (S/D). In the present study, the settlement criterion is selected to be 10% of pile diameter.

Table 4 Soil properties used in case 2

Parameter	E (MPa)	ν	ϕ' (deg)	ψ (deg)	K_0	γ (kN/m^3)
Sand	70	0.3	35	5	0.43	18

Table 5 Pile and soil parameters used in numerical analysis

Parameters	Selected values				
D (m)	0.8	1.2	1.6	–	–
L (m)	10	20	30	–	–
E (MPa)	20	40	70	100	–
ν	0.2	0.3	0.4	–	–
γ (kN/m^3)	15	18	20	–	–
ϕ' (deg)	25	30	35	40	45

5.2 Effect of Pile Diameter and Soil Poisson's Ratio

Figure 5 shows the effect of pile diameter (D) ranging from 0.8 to 1.6 m on end bearing capacity for a series of analyses with different values of friction angle, elastic modulus of soil, and other parameters. Each series of analyses is performed on different sand states shown in Table 6. In addition, the effect of Poisson's ratio (ν) on end bearing capacity is shown in Fig. 7 and properties of pile and soil used to study this parameter are listed in Table 7.

Figure 5 shows that the tip resistance increases with an increase in pile diameter. This is in agreement with the results obtained by other researchers (Brezantzev *et al.* 1961; Han *et al.* 2017; Tolooiyan and Gavin 2015; Ahmadi and Khabbazian 2009). For example, Fig. 1 shows that Brezantzev *et al.* (1961) suggested that the value of ω would be increased when the ratio of L/D decreases. Thus the larger values of pile diameter result in larger values of ω, and this leads to larger tip resistance values. Table 6 shows that for loose and medium dense sands, the percentage increase in tip resistance due to increase in pile diameter is

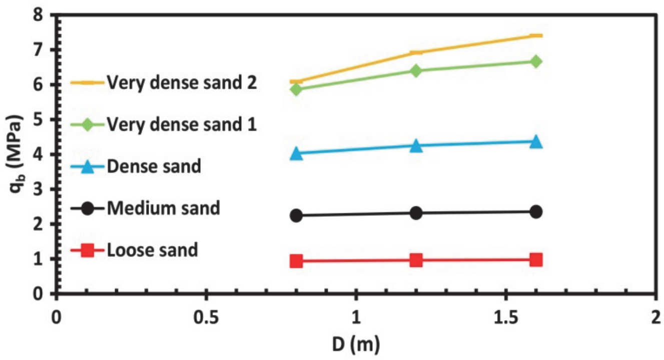

Fig. 5 Effect of pile diameter on tip resistance

Table 6 Soil and pile properties used to study the effect of pile diameter

Type of soil	L (m)	E (MPa)	ν	γ (kN/m³)	ϕ′ (deg)	% increase in q_b due to variation in D
Loose sand	10	20	0.3	15	25	3.9
Medium sand	10	40	0.3	18	35	4.9
Dense sand	10	70	0.3	20	40	8.5
Very dense sand 1	10	100	0.3	20	45	13.6
Very dense sand 2	30	100	0.3	20	45	21.7

Table 7 Soil and pile properties to study the effect of Poisson's ratio

Type of soil	D (m)	L (m)	E (MPa)	γ (kN/m³)	ϕ′ (deg)	% increase in q_b due to variation in ν
Loose sand	0.8	10	20	15	25	21.0
Medium sand	0.8	10	40	18	35	23.4
Dense sand	0.8	10	70	20	40	22.7
Very dense sand	0.8	10	100	20	45	21.3

less than 10%. Therefore, eliminating the effect of pile diameter on end bearing capacity at these density states can be considered acceptable. Meyerhof suggested another method (Eqs. (2) and (5)), referring that end bearing capacity decreases with an increase in the amount of pile diameter and soil Poisson's ratio. The results obtained in this study indicate that the effect of pile diameter on tip resistance values in very dense sand is larger than those for loose sand. Han *et al.* (2017) and Ahmadi and Khabbazian (2009) have also found similar results. This means that the effect of pile diameter should be considered for very dense sand states. Moreover, the longer the pile length, the larger the effect of pile diameter on end bearing capacity. Han *et al.* (2017) also show a similar conclusion in their investigation. In order to study the coupling effect of length and diameter of pile, the tip resistances of some drilled shafts with diameters of 0.8, 1.2 and 1.6 m and lengths of 10, 20 and 30 m embedded in medium dense sand ($\phi' = 35°$, $E = 40$ MPa, $\gamma = 18$ kN/m³, $\nu = 0.3$) are modeled, and the results are shown in Fig. 6. This figure shows that for each value of pile length, tip resistance increases with an increase in the coupling effect of pile diameter and pile length ($\tan^{-1}(D/L) \times \gamma L/P_a$), where P_a is the atmospheric pressure (100 kPa). Also, this figure shows that the increase in pile length leads to an increase in tip resistance. Figure 7 indicates that by doubling the soil Poisson's ratio (ν) from 0.2 to 0.4, the percentage increase in the end bearing capacity is about 22%. This result has also been reported by Ahmadi and Khabbazian (2009).

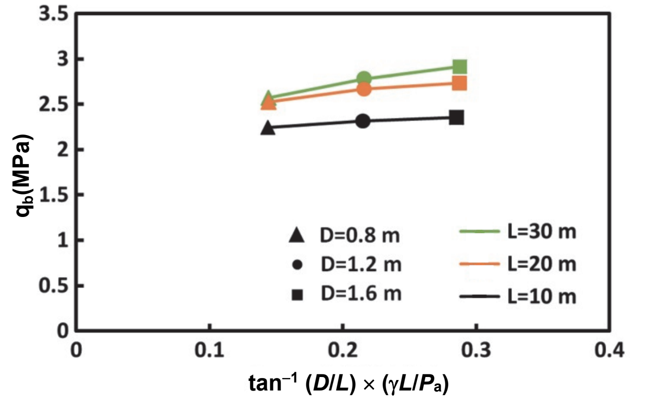

Fig. 6 The coupling effect of pile diameter and length on end bearing capacity (q_b)

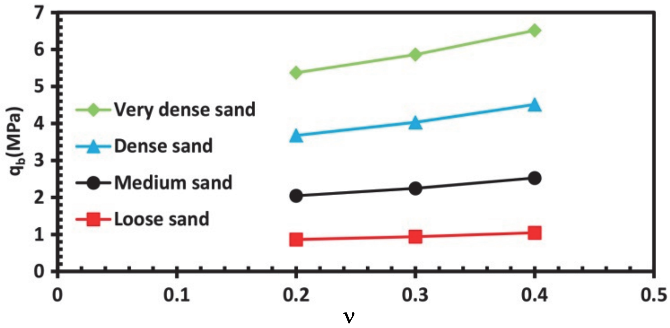

Fig. 7 Effect of Poisson's ratio on tip resistance

5.3 Proposing a New Equation for the Determination of End Bearing Capacity

By analyzing more than 1500 numerical models, the authors suggest Eq. (22) to estimate the end bearing capacity of drill shafts in sand. The parameters in Eq. (22) are already defined, and K_E is given in Eq. (23). In Eqs. (22) and (23), P_a is the atmospheric pressure (100 kPa) and ϕ′ is friction angle in radian. In these equations, drained friction angle and elastic modulus of the soil can be determined from triaxial loading tests in the laboratory. Also, these parameters can be determined from CPT or SPT test measurement through correlations suggested by several researchers (*e.g.,* Robertson and Campanella 1983, Schmertmann 1975). Table 9 presents the typical ranges of geotechnical parameters for each states of sand. Equation (22) shows that pile dimensions have direct effect on end bearing capacity. This means that an increase in D appears in the numerator of tan term resulting in an increase in end bearing capacity. Also, the parameter L appears in the denominator of the tan term, and it also appears in the numerator of Eq. (22). All in all, the parameter L is directly related to end bearing capacity. Furthermore, tip resistace increases with increase in geotechnical parameters (γ′, ν, E, and ϕ′). Figure 8 shows the comparison of pile tip resistance obtained in numerical analyses with values obtained via suggested Eq. (22). Figure 8 shows that calculated values from Eq. (22) are in good agreement with results from numerical modeling. The maximum error band for Eq. (22) is about 20%, which can be considered acceptable.

$$q_b = \gamma' L \times e^{4.7\phi'} \times K_E^{1.2\phi'} \times \left(1 + 0.75\,\phi' \tan^{-1}\left(\frac{D}{L}\right) \times \frac{\gamma' L}{P_a}\right) \le 5000\text{kPa} \tag{22}$$

where

$$K_E = \frac{E}{1000 \times (1-\nu) \times (\gamma' L \times \tan(\phi'))} \tag{23}$$

In order to propose these equations, numerous regression-based and trial and error analyses are performed to obtain an

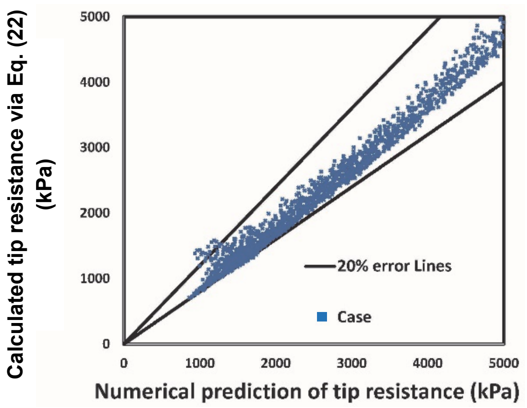

Fig. 8 Analyzed values of tip resistance versus Results of Eq. (22)

equation with suitable factors and their coefficients. The terms of $\gamma'L$, K_E, and $(1 + 0.75\phi'\tan^{-1}(D/L) \times \gamma'L/P_a)$ used in Eq. (22), are basically similar to what Meyerhof (1953), Vesic (1973), and Hansen (1970) have suggested, respectively. It should be noted that the effect of Poisson's ratio (ν) and pile diameter (D) need to be corrected according to the results of Section 5.2. This means that based on Vesic (1973) and Hansen (1970), the tip resistance decreases with an increase in Poisson's ratio (ν) and pile diameter (D), however, results of this study in Section 5.2 shows that tip resistance increases with increase in ν and D.

Also, Salgado (2008) suggested that the tip resistance of drilled shafts be capped with 5 MPa. This limit on tip resistance provides insurance against a number of things that could go wrong in the process of installation (such as the accumulation of loose cuttings on the tip of drilled shafts). It also indirectly accounts for the excessively high values of tip resistance that might result when relative density, and thus SPT and CPT values are very large. This limit value of 5 MPa has also been used in this study. It should be noted that Fig. 8 presents the tip resistances which are lower than 5 MPa.

5.4 A Numerical Example of the Proposed Equation

A centrifuge load test has been introduced in Section 3.2. The properties of this case are given in Table 4. Therefore, the values of drained friction angle, soil elastic modulus, soil unit weight and Poisson's ratio are 35°, 70 MPa, 18 kN/m^3, and 0.3, respectively. Also, diameter and length of the drilled shaft are 0.3 and 8 m, respectively. With these assumptions, Eqs. (23) and (22) are equal to:

$$K_E = \frac{E}{1000\times(1-\nu)\times(\gamma' L\times\tan(\phi'))}$$

$$= \frac{70\times10^6}{1000\times(1-0.3)\times(18\times10^3\times8\times\tan(35°))} = 0.99$$

$$q_b = \gamma' L\times e^{4.7\phi'}\times K_E^{1.2\phi'}\times\left(1+0.75\,\phi'\tan^{-1}\left(\frac{D}{L}\right)\times\frac{\gamma' L}{P_a}\right)$$

$$= 18\times10^3\times8\times e^{4.7\times35\times\frac{3.14}{180}}\times(0.99)^{1.2\times35\times\frac{3.14}{180}}$$

$$\times\left(1+0.75\times35\times\frac{3.14}{180}\times\tan^{-1}\left(\frac{0.3}{8}\right)\times\frac{18\times10^3\times8}{10^5}\right)$$

$$= 2582 \text{ kPa}$$

It should be noted that the value of $\tan^{-1}(0.3/8)$ must be in radian.

6. PERFORMANCE CHECK OF THE SUGGESTED EQUATION

The authors collected some measured data of drilled shafts embedded in sand which are shown in Table 8. These measured data are associated with a settlement of 10% of pile diameter.

In order to investigate the suitability of Eq. (22) in determining the tip resistance, measured data presented in Table 8 are compared with the results obtained using the suggested equation. These measured data are also compared with other methods available in the literature (relationships based on geotechnical parameters, some CPT and SPT correlations). All of these methods will be disscussed in sections 6.1, 6.2, and 6.3. The results of the comparisons are shown in Fig. 9. In this figure the values of measured data against predicted values via Eq. (22) are bounded by the lines of ±20% error. In addition, the values of the mean absolute percentage error (MAPE) are also calculated for each method and are shown in Fig. 9. The MAPE value (Eq. (24)) is a measure of prediction accuracy of a forecasting method in statistics, for example in trend estimation.

$$\text{MAPE} = \frac{1}{n}\times\sum_{t=1}^{n}\left|\frac{(q_{b\text{-measured}} - q_{b\text{-calculated}})}{q_{b\text{-measured}}}\right|\times100 \tag{24}$$

where n is the number of measured data.

6.1 Approach 1: Relationships Based on Geotechnical Parameters

As it was discussed in introduction of this paper, Meyerhof's and Brezantzev's methods are two well-known relationships based on geotechnical parameters available in the literature. These relationships are based on the geotechnical parameters of soil and dimensions of drilled shaft. Another well known equation was proposed by Salgado (2008) in which soil geotechnical parameters are used. Equation (22) recommended in this study includes these parameters as well. These equations use geotechnical parameters and measured values presented in Table 8 cotaining only soil friction angle. Therefore, a correlation between friction angle (ϕ') and soil elastic modulus (E), and a correlation between friction angle (ϕ') and soil unit weight (γ), and a correlation between friction angle (ϕ') and soil relative density (D_r) are required. Chen and Kulhawy (1994), Carter and Bentley (2016), and Terzaghi and Peck (1948), among others, have suggested empirical relationships for elastic modulus, unit weight, and friction angle based on different states of sands, respectively. Table 9 illustrates the ranges of these geotechnical parameters based on the suggestions of these researchers. It should be noted that Table 9 provides a general

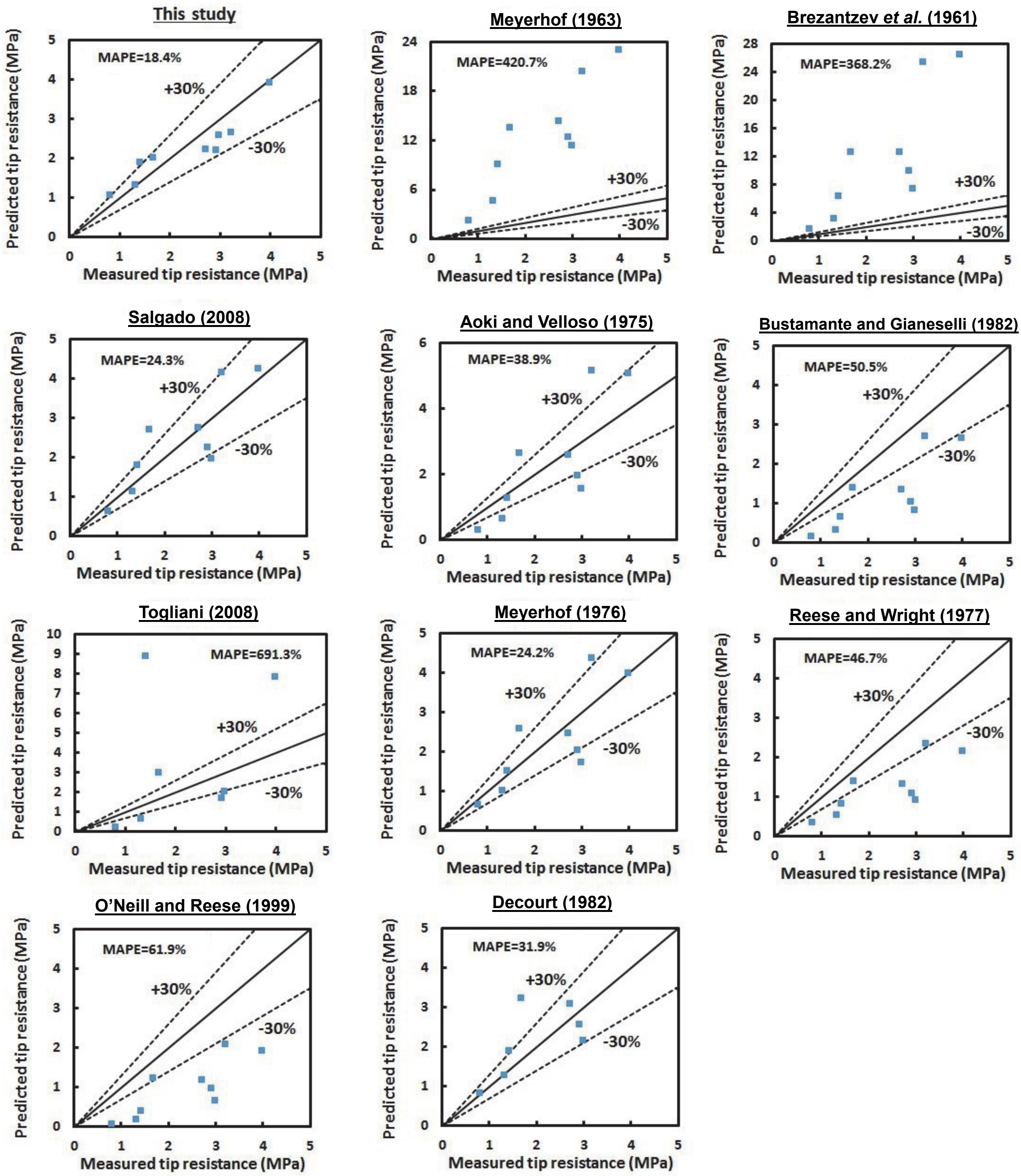

Fig. 9 Comparison of different methods with measured field data

estimation of parameters, and it is possible that some samples of soils have larger or smaller values of these parameters. An important issue that can affect the values of these parameters is the soil compaction and relative density (D_r). For example, a sample of sand with a friction angle of 36° and a large value of D_r such as 90% ~ 100% can have an elastic modulus of about 70 ~ 90 MPa. Therefore, the soil elastic modulus should be determined by laboratory tests, but in this study due to the inexistence of results of laboratory tests, Table 9 is used by considering the engineering judgment.

Ahmadi and Khabbazian (2009) used a correlation between soil friction angle and soil relative density (Eq. (25)). This equation is used to correlate relative density of sand used in Salgado's method with soil friction angle.

$$\phi = 25 \exp(0.7\,D_r) \tag{25}$$

Table 8 Field measurement data collected by Yasufuku *et al.* (2001)

Case	Reference	D (m)	L (m)	ϕ' (deg)	σ' (kPa)	q_b-measured (kPa)
1	BCP (1971)	0.2	4	35	60	1300
2	JGS data (1993)	1.5	22.4	35	180	2900
3	Kyushu Branch of JGS (1997)	1.2	41	38	280	3980
4	Yasufuku *et al.* (2001)	-[a]	5	36	100	1400
5	Yasufuku *et al.* (2001)	-[a]	10	36	200	2700
6	Yasufuku *et al.* (2001)	-[a]	20	36	400	3200
7	Mayne and Harris (1993)	0.76	16.8	34.8	250	1660
8	Fioravante (2002)	0.3	8	35	144	2970
9	Mullins *et al.* (2001)	0.216	2.44	33	39	800

[a]: Physical model has been conducted, and length has been calculated from vertical stress

Table 9 Range of geotechnical parameters for sand

Sand type	Dry unit weight (kN/m^3)	Friction angle (deg)	Elastic modulus (MPa)
Loose sand	13 ~ 15	< 30	10 ~ 20
Medium dense sand	15 ~ 18	30 ~ 36	20 ~ 50
Dense sand	18 ~ 22	> 36	50 ~ 100

6.2 Approach 2: CPT Correlations

As discussed in Section 2.1, some correlations have been provided in the literature to estimate tip resistance of drilled shafts embedded in sand based on CPT values. This study evaluates the performance of the methods of Aoki and Velloso (1975), Bustamante and Gianeselli (1982), and Togliani (2008) which are Eqs. (11) to (13), respectively. It should be noted that Table 8 is based on soil friction angle. Therefore, the correlation of Robertson and Campanella (1983) (Eq. (14)) is used in this approach to correlate q_c values to soil friction angle.

6.3 Approach 3: SPT Correlations

The end bearing capacity of drilled shafts in sandy soils is generally estimated by using cone penetration test (CPT) or standard penetration test (SPT) results obtained at the site where drilled shafts are constructed.

As discussed in Section 2.2, some correlations have been provided in the literature to estimate tip resistance of drilled shafts embedded in sand based on SPT values. This study uses the methods of Meyerhof (1976), Reese and Wright (1977), Decourt (1982), and O'Neill and Reese (1999) which are Eqs. (15) to (18), respectively. As it was mentioned before, Table 8 is based on soil friction angle. Therefore, the correlation of Kulhawy and Mayne (1990) (Eq. (20)) is used in this approach to correlate N values to soil friction angle.

6.4 Discussion on the Results of Different Approaches

Figure 9 shows that among the relationships based on geotechnical parameters suggested in the literature. The equation suggested in this study (Eq. (22)) provides a better estimation of tip resistance. It should be noted that Eq. (22) predicts the value of tip resistance associated with a settlement of 10% of pile diameter, but Meyerhof's and Brezantzev's methods are based on the limit equilibrium analysis. As mentioned in the introduction section of this paper, the limit equilibrium analysis calculates the ultimate tip resistance which needs a very large displacement. It was mentioned that the load-settlement curve of drilled shafts in sand increases without any peak point for settlement values associated with 20% of pile diameter. (*e.g.,* Figs. 3 and 4). Therefore, the value of tip resistance for a larger settlement is greater than smaller one, and this is the reason of high value of Meyerhof's method and Brezantzev's equation. On the other hand, the Salgado's method was proposed to estimate tip resistance associated with a settlement of 10% of pile diameter. As Fig. 9 shows, this method can acceptably estimate the tip resistance with the MAPE value of 24.3%.

Figure 9 also shows that the method of Aoki and Velloso (1975) using q_c values in their suggested equation can lead to acceptable results, if Robertson and Campanella (1983) equation is used to correlate the soil parameters measured in the field to CPT values. Furthermore, this results indicate that among the SPT based methods, the relationships of Meyerhof (1976) and also Decourt (1982) using the SPT values in their equation can lead to acceptable results, if Kulhawy and Mayne (1990) equation is used to correlate the soil parameters measured in the field to SPT values. Relationships of Reese and Wright (1977) and O'Neil and Reese (1999) are also suggested for design of pile, and these relationships underestimate tip resistances as can be seen from Figure 9.

In Fig. 9, the value of MAPE for Togliani (2008) method is high. This value of MAPE resulted from Cases 4, 5, and 6. These cases are the results of physical model tests performed by Yasufuku *et al.* (2001) and the values of pile length are calculated from vertical stress with the assumption of 18 kN/m^3 for soil unit weight. The values of tip resistance for cases 5 and 6 predicted by Togliani's method are 34.4 MPa and 135.8 MPa, respectively. These values are not shown in Fig. 9 since they are too large to be fitted in the figure. Therefore, the authors removed the cases 4, 5, and 6, and recalculated the value of MAPE for Togliani's method. The new value of MAPE is 56.3%, and this shows that Togliani's method gives a better but still unsatisfactory estimation for tip resistance.

Also, it should be noted that correlations used to correlate the values of soil friction angle to q_c and N have an important role in the results of two CPT and SPT approaches. It is evident that making use of different correlations can lead to different results for these two approaches. The suggested equation in this study results in the lowest value for MAPE (*i.e.,* 18.4%) among other methods (Fig. 9).

6.5 Limitation and Application

Most of the methods proposed to estimate tip resistance of drilled shafts in sand are based on SPT or CPT values. Equation (22) proposed in this study is based on soil geotechnical parameters which can acceptably estimate tip resistance of drilled shafts in sand. However, there are some limitations to this method. Therefore, in order to use this method, these limitations should be considered.

- Equation (22) is proposed for homogeneous sandy soils continuing for a sufficient distance from pile tip. This distance must be larger than the plastic zone occurring around the pile tip. More details about the plastic zone around the pile tip are beyond the scope of this paper. Therefore, this method should not be used when the soil beneath the pile tip is layered. Also, this method is only valid for sandy soils, and is not applicable for such soils as silty sands or cemented sands.
- This method is proposed based on the range of parameters given in Table 5. Therefore, when the values of parameters for the soil are not in the range given in Table 5, further investigation for its applicability is warranted.
- Measured field results shown in Table 8 show that this method can be used for pile length between 2.5 and 41 m, and pile diameter between 0.2 and 1.5 m.
- The method proposed in this study is based on geotechnical parameters. Therefore, some laboratory tests should be performed to determine the geotechnical parameters (*e.g.,* soil friction angle and elastic modulus). Determination of these parameters (specially soil elastic modulus) is a debatable issue in geotechnical engineering.

7. CONCLUSIONS

In this study, the end bearing capacity of drilled shafts, embedded in sand, is investigated. The FLAC software, a program based on finite difference method, is used to model drilled shafts in sand. After verification of the models, a series of analyses was conducted. The result of these analyses led to the following conclusions:

1. An increase in pile diameter leads to an increase in tip resistance of drilled shafts in sandy soils, especially for dense sands. The percentage of this increase is higher for longer piles. By doubling the value of the pile diameter from 0.8 to 1.6 m, tip resistance can be increased more than 20 percent.
2. End bearing capacity is increased by an increase in the value of Poisson's ratio (ν). The parametric study shows that the effect of Poisson's ratio is more than 20 percent when it varies from 0.2 to 0.4.
3. A new equation (Eq. (22)) is proposed to estimate the value of tip resistance. It has an acceptable agreement with the field data.
4. Some proposed equations based on geotechnical parameters, CPT and SPT methods are compared with the field measurement results. The results show that the equation proposed in this study provides better prediction of tip resistance compared to other methods based on geotechnical parameters. The method of Aoki and Velloso (1975) using q_c values in their suggested equation can lead to acceptable results, if Robertson and Campanella (1983) equation is used to correlate the soil parameter measured in the field to q_c values. Furthermore, this results indicate that among the SPT based methods, the relationships of Meyerhof (1976) and also Decourt (1982) using the SPT values in their equation can lead to acceptable results, if Kulhawy and Mayne (1990) equation is used to correlate the soil parameter measured in the field to SPT values. Salgado's equation which is based on geotechnical parameters results to acceptable values of tip resistance, too.

NOTATIONS

D	Pile diameter (m)
D_r	Relative density (%)
E	Elastic modulus (MPa)
F_{qc}	Compressibility factor
F_{qd}	Depth factor
F_{qs}	Shape factor
I_r	Rigidity index
I_{rr}	Reduced rigidity index
K_E	Elastic modulus factor
K_0	Coefficient of lateral Earth pressure at rest
K_n	Normal stiffness (N/m^3)
K_s	Shear stiffness (N/m^3)
L	Pile length (m)
N	SPT value
N_q	Bearing capacity factor
N^*_q	Bearing capacity factor
P_a	Atmospheric pressure (Pa)
q_b	End bearing capacity (Pa)
q_c	Cone tip resistance (Pa)
$q_{c(\text{tip})}$	Cone tip resistance at pile base (Pa)
$q_{ca(\text{tip})}$	Arithmetic average of q_c around the pile base (Pa)
q_{t1}	Corrected q_c to yield (Pa)
q'	Effective vertical stress at the base of the shaft (Pa)
S	Settlement (mm)
γ	Unit weight (kN/m^3)
γ'	Effective unit weight (kN/m^3)
ν	Poisson's ratio
σ'_m	Mean effective stress (Pa)
σ'_{v0}	Effective vertical stress (Pa)
ϕ'	Drained friction angle (°)
ϕ_{int}	Soil friction angle (°)
ψ	Dilation angle (°)
ω	Correction factor

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