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UPPER BOUND ANALYSES OF PULLOUT CAPACITY OF STRIP ANCHORS IN UNDRAINED SLOPING CLAYEY GROUND

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ABSTRACT

Anchors are mainly employed in foundation systems for resisting uplift forces. A variety of approximate techniques have been proposed to estimate the ultimate pullout capacity of strip anchors in clay. However, little information is available to calculate the pullout capacity of strip anchors installed in a sloping ground. This study presents upper bound analyses of pullout capacities and the corresponding failure surface for strip anchors in a sloping clayey ground. Comparisons of these upper bound solutions with the laboratory test results obtained by Khing *et al.* & Rao and Prasad are carried out. It is indicated that the laboratory test results tend to underestimate the reduction of pullout capacity of anchors due to the presence of sloping ground, which may introduce safety issues when used for practical designing. The discrepancy between our theoretical results and the laboratory test results is shown, and possible explanations for it are discussed. Simple equations for predicting the pullout capacity of strip anchors are developed to facilitate the use of strip anchors in solving practical problems.

Key words: Anchors, clay, sloping ground, failure surface, pullout capacity, limit analysis.

1. INTRODUCTION

Plate anchors are widely applied in foundation systems for structures requiring uplift resistance such as earth-retaining walls, transmission towers and mooring systems for offshore floating facilities. As required by practical engineering design, anchors can be installed by various techniques in a sand or clay ground, and also can be in various shapes depending on intended purpose. A variety of studies on predicting pullout capacity of strip anchors are available in the literature, most of which have been developed based on curve-fitting data obtained from laboratory model tests (Das and Seeley 1975; Rowe and Davis 1982a; Rowe and Davis 1982b; Frydman and Shaham 1989; Murray and Geddes 1989; Khing *et al.* 1994; Dickin and Laman 2007). In terms of theoretical solutions, many studies involve the use of limit equilibrium method (Meyerhof and Adams 1968; Choudhury and Subba Rao 2005; Choudhury and Subba Rao 2007; White *et al.* 2008), even though *a priori* assumption on failure surface must be made to conduct calculation of collapse load. Other solutions have also been developed by using the finite element method (Wang *et al.* 2010; Yang *et al.* 2010; Yu *et al.* 2011; Cassidy *et al.* 2012; Yang *et al.* 2012; Tho *et al.* 2013), cavity expansion theory (Yu 2000), and limit analysis (Regenass and Soubra 1995; Kumar 1997; Merifield *et al.* 2001; Merifield 2002; Merifield and Sloan 2006).

However, prediction of pullout capacity of anchors installed in a clayey ground with sloping surface is rarely conducted, though many studies have been undertaken for anchors in a clayey ground with horizontal surface. With exception of Khing *et al.* (1994) & Rao and Prasad (1992), who reported some results of laboratory model tests, no rigorous study using limit analysis method has been undertaken. In this paper, our attention is focused on the case of strip anchors in a undrained sloping clayey ground, for which a plane strain condition can be assumed if the length of the anchor under consideration is significantly larger than its width. A wide range of slope surface angles and anchor embedment ratios are investigated. Comparisons with the laboratory test results of Khing *et al.* (1994) & Rao and Prasad (1992) are particularly discussed and the discrepancy between our theoretical results and the laboratory test results is shown. Possible explanations for this discrepancy are discussed even though we cannot prove it at the current stage. Simple analytical equations for predicting the pullout capacity of strip anchors are developed, which would be useful to estimate the pullout capacity of strip anchors installed in a sloping ground. This study is expected to deepen our understanding of pullout behaviors of anchors installed in a sloping clayey ground and promote the awareness of the discrepancy between the results of our theoretical analysis and published laboratory test.

2. DEFINITION OF PROBLEM

A schematic of the problem considered in this study is shown in Fig. 1. Anchors can be classified as shallow or deep ones, depending on their modes of failure. As shown in Fig. 1, the failure surface for shallow anchors reaches the sloping ground surface at ultimate collapse. In contrast, the failure mode for deep anchors is characterized by localized shear around the anchor itself and is independent of the location of the ground surface. Thus, when the deep failure mode occurs, the ultimate capacity of strip anchor will reach a maximum limiting value. As depicted in Fig. 1, Q_u is the ultimate load of strip anchor; B is the width of anchor plate; β is an acute angle between the sloping ground surface and the horizontal direction; H is the depth of anchor embedment, which is defined as the vertical distance from the midpoint of anchor plate to the ground surface; and H_{cr} is the critical anchor embedment depth below which the failure mechanism no longer extends to the ground surface and becomes fully localized around the plate anchor. As shown in Fig. 1, the

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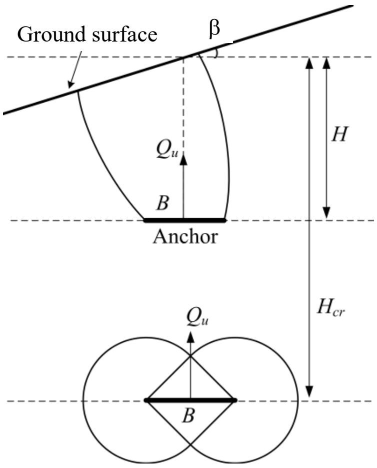


Fig. 1 **Problem notation: deep and shallow anchors in a sloping ground**

direction of pullout force is assumed to be perpendicular to the anchor face, and the plate anchor is taken to be rigid and have zero thickness. The clay is taken to be purely cohesive for undrained condition and thus the undrained shear strength of clay c_u is the only strength parameter to be considered.

The ultimate pullout capacity of strip anchors in purely cohesive soil can be expressed as a function of the undrained shear strength in the following form

$$q_u = \frac{Q_u}{B} = c_u N_c \tag{1}$$

where N_c is the pullout factor of strip anchor.

3. KINEMATICALLY ADMISSIBLE VELOCITY FIELD BASED ON BLOCK SET MECHANISM

This study follows the authors' work on block set mechanism (Yu 2011; Yu *et al.* 2014). The exactly same upper bound mechanism is used here and shown in Fig. 2, in which a large number of sliding triangular rigid blocks are arranged in a particular way and separated by infinitesimally thin regions undergoing shear (Chen 1975). It can be seen that the kinematically admissible velocity field is constructed using four basic block sets. The basic block set with a polar coordinate system is also shown in Fig. 2, where the point O is the pole and ρ_i is the radius. θ_1 and θ_2 are the start angle and end angle of basic block set. The total internal energy E is given by the sum of each internal energy of all velocity discontinuities. The total work done by the weight of soil (W) can also be obtained by the sum of the work done by the soil weight of all rigid blocks. The work done by the pullout force Q_u is

$$F = Q_u v_0 \tag{2}$$

where v_0 is the velocity of the strip anchor.

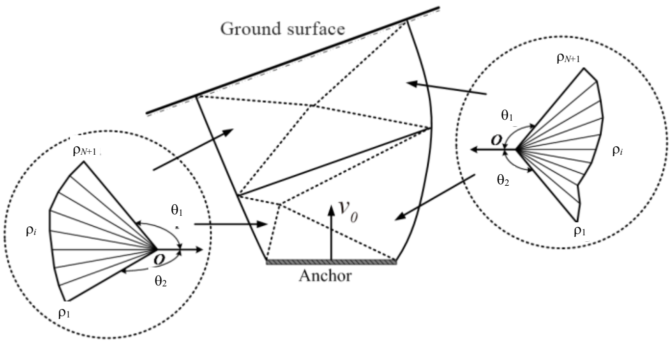


Fig. 2 **Construction schematic of admissible velocity fields for anchor in clay with a sloping surface**

According to the upper bound theorem (Chen 1975), an upper bound solution of pullout capacity of plate anchors can be obtained by solving the equation

$$F + W = E \tag{3}$$

Substituting Eq. (3) and Eqs. (2) to (1) leads to the following expression,

$$N_c = \frac{E - W}{B v_0 c_u} \tag{4}$$

As the pullout factor of strip anchor predicted by Eq. (4) is an upper bound solution, one more step needs to be conducted to find the geometry of the upper bound mechanism that gives a minimal pullout factor of strip anchor. Interior angles and edge lengths of triangular rigid blocks are treated as unknowns and a hybrid genetic algorithm combined with a pattern search method (or direct search method) (MATLAB 2011) is used to optimize these variables. The number of triangular rigid blocks adopted in the mechanism is determined adaptively in order to provide enough accuracy. The final mechanism may include hundreds of triangular rigid blocks. Full details of block set mechanism can be found in Yu *et al.* (2014).

4. RESULTS AND DISCUSSION

Given the fact that the pullout capacity of deep anchor is a constant value and a variety of studies on pullout capacities of horizontal anchor have been published, these two case are illustrated first as a benchmark example to verify the effectiveness of the proposed upper bound mechanism. Then the discrepancy between our theoretical results and the laboratory test results of Khing *et al.* (1994) & Rao and Prasad (1992) is discussed and possible explanations for this discrepancy are proposed. At last simple analytical equations for predicting the pullout capacity of strip anchors are developed for further use of this work.

4.1 Verification of Proposed Upper Bound Mechanism

The pullout factors of deep anchors computed by the upper bound solutions of Rowe (1978) and by the block set mechanism in current study are shown in Table 1, where N is the number of

Table 1 Results of deep anchor with perfect rough condition

N	N_c	Error with Rowe (1978) (%)	Upper bound solution of Rowe (1978)
6	12.5551	9.8943	$3\pi + 2$ (11.4248)
10	11.6993	2.4036	
18	11.4930	0.5974	
34	11.4418	0.1491	
66	11.4290	0.0372	
130	11.4258	0.0093	
258	11.4250	0.0023	
514	11.4249	0.0011	
1026	11.4248	0.0002	

triangular rigid blocks. Comparisons of the failure surface predicted by both methods are also shown in Fig. 3. For clarity, the triangular rigid blocks of the block set mechanism are denoted by circles. Figure 3 indicates that, for deep anchors, the upper bound solution obtained in this study is consistent with analytical solution of Rowe (1978).

Gunn (1980) used the bound theorems to find solutions for the case of a horizontal strip anchor and trapdoor. The three-variable block mechanism proposed by Gunn is shown in Fig. 4. For the purpose of comparison, the failure surfaces at different embedment ratios (H/B) obtained from both the current study and Gunn's mechanism are shown in Fig. 5. It can be seen that this study predicts a bigger failure surface for greater embedment ratios ($H/B > 8$).

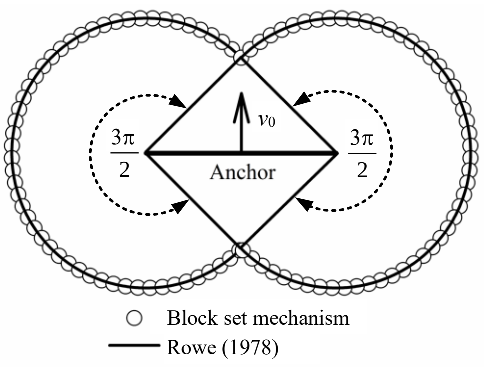


Fig. 3 Comparison of failure mechanism with Rowe (1978)

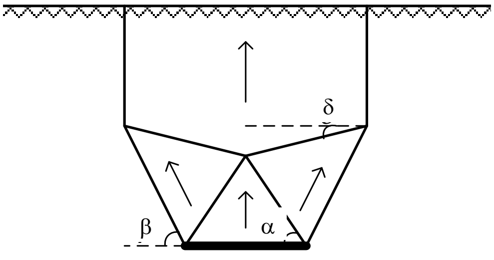


Fig. 4 Gunn's upper bound mechanism

For anchors installed in horizontal ground, comparisons of pullout capacities from the current study against existing upper bound solutions are shown in Fig. 6, where H_{c0} is the pullout factor of strip anchor without considering soil weight. It can be seen that the solutions of this study are between those of Gunn (1980) and Merifield *et al*. (2001). For anchors at small embedment ratios ($H/B < 4$), these upper bound solutions seem consistent.

4.2 Comparisons with Laboratory Test Results

As part of a foundation system, anchors may also be employed in sloping terrain conditions such as sea bed or mountainous area. In such case, it is essential to evaluate the effect of slope angle on the pullout capacity of anchor, especially considering that the presence of slope can lead to a lower pullout capacity of plate anchors than that of a horizontal ground condition. Comparisons of pullout capacities from the current study and laboratory model tests by Khing *et al*. (1994) are shown in Fig. 7. It can be seen that the current study predicts a significantly reduction in pullout capacity when the slope angle β varies from 0° to 25°.

Another series of laboratory model tests were conducted by Rao and Prasad (1992) at slopes of 0°, 15°, 30°, and 45°. Since the model tests were carried out on a 50 mm diameter model plate anchor, the shape factor, which is a ratio of pullout factors for circle and strip anchors, must be taken into account. Due to the consistency of predictions between the current study and model test for the case of horizontal ground (as indicated in Fig. 7), the shape factors can be estimated from the ratio of

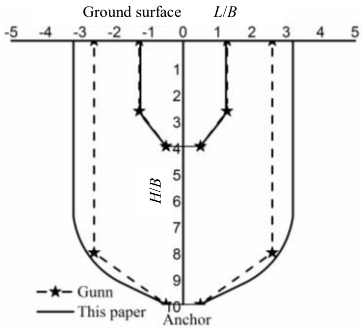


Fig. 5 Comparison of failure surface between Gunn (1980) and the current study

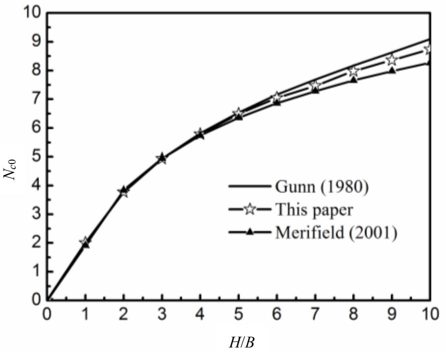


Fig. 6 Comparison of pullout capacity of anchors in horizontal ground with existing upper bound solutions

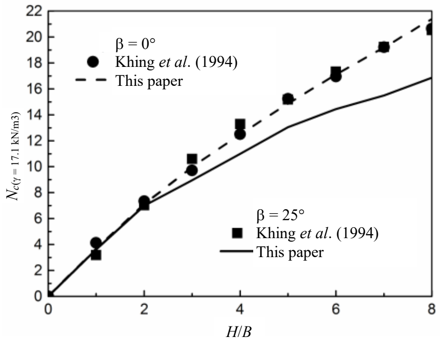


Fig. 7 Comparison of pullout capacity of plate anchors with model test of Khing *et al*. (1994)

anchor pullout capacity measured from model tests of Rao and Prasad (1992) to that theoretically predicted in the current study. For convenience, the effect of slope angle on shape factor is not considered and the same shape factors are adopted at all slope angles. The shape factors used in the current study can be expressed by Eq. (5). Comparisons of pullout capacities obtained from the current study and laboratory model tests of Rao and Prasad (1992) are shown in Fig. 8. It can be seen that both approaches show similar prediction accuracies for anchors of small embedment ratios ($H/B < 3$). But for anchors of larger embedment ratios, the laboratory model test results show that the reduction of pullout capacity decreases with an increase of embedment ratio H/B.

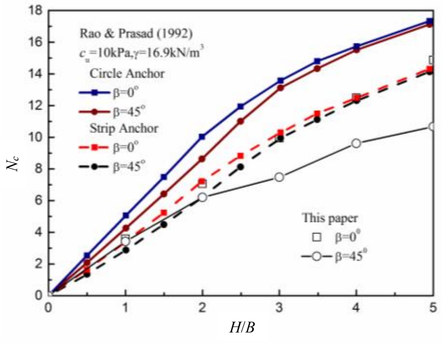
$$S = 1.7 - 0.22\sqrt{\frac{H}{B}} \tag{5}$$

If $S < 1.15$, $S = 1.15$.

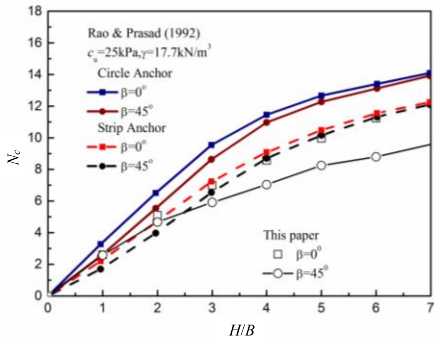
Although both Khing *et al*. (1994) & Rao and Prasad (1992) indicated that the pullout capacity decreases with an increase of β, the results of model tests seem to underestimate the reduction of pullout capacity for larger anchor embedment ratios. The discrepancy between our theoretical analysis and these experimental results is significant. It would be unsafe for a design based on these laboratory test results, since which overestimates the pullout capacity of anchor installed in a sloping ground. The authors wish to show this discrepancy here and promote the awareness of this issue. Though we cannot prove it, the following is our explanation on the discrepancy and we will conduct 3D analysis in our future work for more proof.

The authors think that this discrepancy between the experimental and theoretical results may be caused by the way of loading in model test arrangement. A schematic diagram of laboratory test arrangement adopted by Khing *et al*. (1994) & Rao and Prasad (1992) is shown in Fig. 9, where a plexiglas box was placed to the underside of the anchor plate to eliminate suction force. So an immediate breakaway case in the model test can be simulated. Due to the presence of the plexiglas box, the failure mode of anchors is restricted to a shallow failure mode, which cannot transform to a deep failure mode. Some of model test results are found to produce an N_c value bigger than 11.42. A steel rod with a specified diameter was welded to the anchor plate for loading purpose.

According to failure surfaces optimized as the minimization of upper bound solution in the current study, the rod is located on the axis of symmetry of failure surface for the horizontal ground case, as shown in Fig. 5 and Fig. 11. When a smooth rod is considered, the skin resistance of the rod can be ignored. But for anchors installed in a sloping ground, as shown in Figs. 10 ~ 11, the rod does not locate on the axis of symmetry of failure surface because the failure surface of anchors in sloping ground tends to incline to the toe of slope. It can be seen from Figs. 10 ~ 11 that the inclination of failure surface towards the horizontal direction increases with increasing anchor embedment ratio and also with increasing slope angle. In this scenario, the normal force induced by the inclination of failure surface tends to act on the skin of rod, which further gives a very appreciable contribution to the pullout capacity of anchors. Hence, the presence of the rod can partially offset the effect of slope angle on the reduction of pullout capacity for anchors. The amount of offset can increase with increasing anchor embedment ratio, which explains the discrepancy for anchors in larger embedment ratios between model test results and the current study. A trapdoor experiment model, which uses no rod in soil, would be a better choice to study the pullout capacity of anchors in sloping ground.



(a) $c_u = 10$ kPa, $\gamma = 16.9$ kN/m^3



(b) $c_u = 25$kPa, $\gamma = 17.7$ kN/m^3

Fig. 8 Comparison of pullout capacities predicted in this study with model tests of Rao and Prasad (1992)

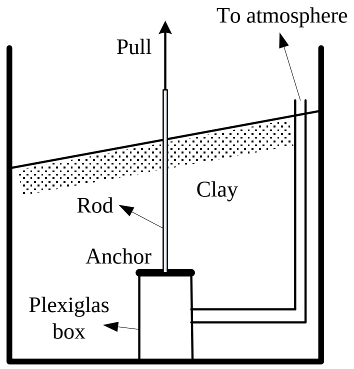
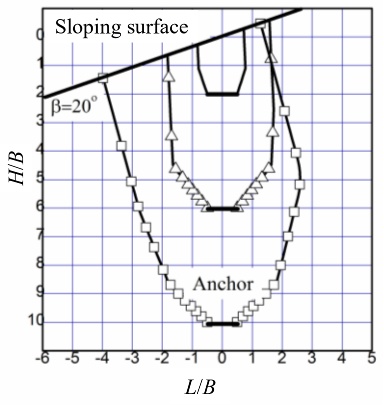
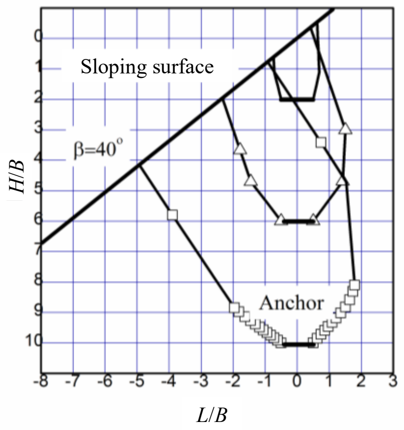


Fig. 9 Schematic diagram of laboratory test arrangement



(a) β = 20°



(b) β = 40°

Fig. 10 Failure surfaces of anchors in various embedment ratios

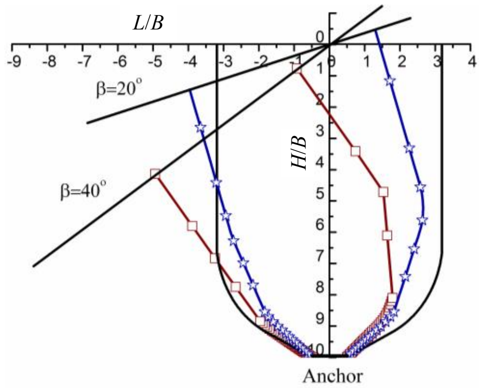


Fig. 11 Failure surfaces of anchors at different slope angles (H/B = 10)

4.3 Simple Equations for Pullout Capacity of Strip Anchors in a Sloping Clayey Ground

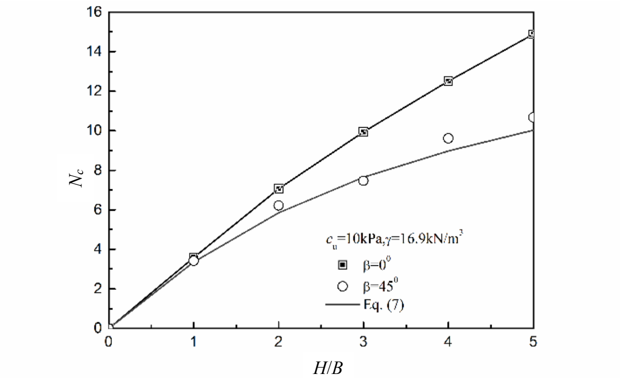
As aforementioned, there is a discrepancy between laboratory test results and theoretical results of the current upper bound limit analysis. However, besides the abovementioned two laboratory test results, little information is available for estimating the pullout capacity of anchors installed in a sloping clayed ground. Meanwhile, an analytical approach using simple equations with few parameters would be efficient to provide design guidelines for engineers. Therefore, simple analytical equations are developed and the upper bound solutions of the current study for anchors in a sloping ground can be best represented by the following equations:

$$N_{c0\beta} = \frac{\pi}{2}\ln\left\{\frac{1}{2}\left[\left(\frac{3\pi}{2}-\beta\right)\left(\frac{H}{B}\right)^2+\tan(\beta)\right]+1\right\} \qquad (6)$$

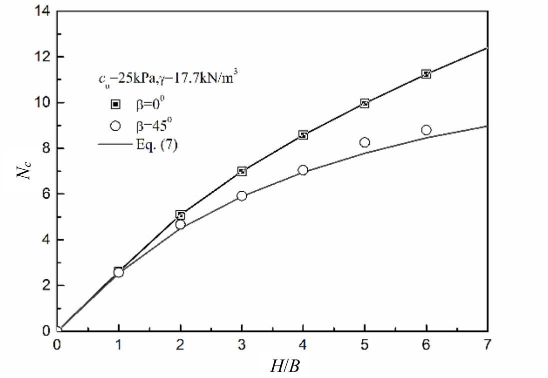
$$N_c = N_{c0\beta} + \frac{\gamma H}{c_u} - 0.08 N_{c0\beta}\frac{\gamma H}{c_u}\tan(\beta) \qquad (7)$$

where (a) if $N_c > 11.42$, the anchor is said to be a deep anchor, and then $N_c = 11.42$; and (b) if $N_c < 11.42$, the anchor is a shallow anchor.

For slope angles varying from 0° (horizontal) to 45°, pullout capacities predicted from the current upper bound analysis and Eq. (7) are presented in Fig. 12. It can be seen that Eq. (7) can properly replicate the results obtained from the current upper bound analysis based on block set mechanism.



(a) c_u = 10 kPa, γ = 16.9 kN/m³



(b) c_u = 25 kPa, γ = 17.7 kN/m³

Fig. 12 Comparison of pullout capacities between the current upper bound analysis and the proposed analytical equations

5. CONCLUSIONS

Upper bound analyses for pullout capacities, as well as failure surfaces of strip anchors in a sloping clayey ground in undrained conditions, were presented. The effect of ground sloping angle on the pullout capacities and failure surfaces of strip anchors was discussed. Comparisons between the current theoretical results and the laboratory test results of Khing *et al.* (1994) & Rao and Prasad (1992) were conducted and the discrepancy was discussed. The current study indicates that the laboratory test results of Khing *et al*. (1994) & Rao and Prasad (1992) tend to underestimate the reduction of pullout capacities of anchors installed in a sloping ground, which will introduce safety problems in practical designing process. Although the authors cannot prove it at the current stage, possible explanations for this discrepancy were proposed. The authors wish to show this discrepancy in this study and promote the awareness of this issue. Simple analytical equations for predicting the pullout capacity of strip anchors were developed, which would be useful to estimate the pullout capacity of strip anchors installed in a sloping ground.

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CONFLICT OF INTERESTS

The authors declare that there is no conflict of interests regarding the publication of this paper.

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